

## Chapter 4

# DEFINITION OF QUANTITIES EVALUATED BY WAMIT

The forces and other quantities evaluated by WAMIT are output in a standard nondimensional form, in terms of the appropriate combinations of the water density  $\rho$ , the acceleration of gravity  $g$ , the incident-wave amplitude  $A$ , frequency  $\omega$ , and the length scale  $L$  defined by the input parameter ULEN in the GDF file. (The volume and coordinates of the center of buoyancy are exceptions. They are output in a dimensional form.)

Body motions and forces are defined in relation to the origin of the same Cartesian coordinate system relative to which the panel offsets are defined. Note that this origin may be located on, above or below the free surface. *If planes of symmetry are defined for the body, the origin must always lie on these planes of symmetry.* The  $x$ - and  $y$ -axes must be parallel to the mean position of the free surface.

The notation and definitions of physical quantities here correspond with those in Reference [3], except that in the latter reference the  $y$  axis is vertical.

All of the evaluated quantities are included with appropriate identification in the OUT output file. To facilitate post-processing the same quantities are also saved in the numeric output files, following the format defined in Section 4.9.

In the cases of field data and mean drift forces (Options 5-9) the definitions given below in Sections 4.5-4.8 apply to the complete solution for the combined radiation and diffraction problems. The same quantities can be evaluated separately for either the radiation or diffraction problems, as explained in Section 4.10.

For the sake of simplicity, the definitions which follow in this Section assume that the origin of the coordinate system is located on the free surface. Special definitions apply to some quantities if vertical walls are defined, as explained in Section 5.3.

## 4.1 HYDROSTATIC DATA

All hydrostatic data can be expressed in the form of surface integrals over the mean body wetted surface  $S_b$ , by virtue of Gauss' divergence theorem.

a) Volume

$$\forall = - \iint_{S_b} n_1 x dS = - \iint_{S_b} n_2 y dS = - \iint_{S_b} n_3 z dS$$

All three forms of the volume are evaluated in WAMIT, as independent checks of the panel coordinates, and printed in the summary header of the output file. The median volume of the three is used for the internal computations. If it is less than  $10^{-30}$ , a warning is displayed and the coordinates of the center of buoyancy are set equal to zero. For bottom-mounted structures, where panels are not defined on the bottom, the last integral defined above differs from the correct submerged volume as noted in Section 3.1.

b) Coordinates of center of buoyancy

$$x_b = \frac{-1}{2\forall} \iint_{S_b} n_1 x^2 dS$$

$$y_b = \frac{-1}{2\forall} \iint_{S_b} n_2 y^2 dS$$

$$z_b = \frac{-1}{2\forall} \iint_{S_b} n_3 z^2 dS$$

c) Matrix of hydrostatic and gravitational restoring coefficients

$$\begin{array}{ll} C(3,3) = \rho g \iint_{S_b} n_3 dS & \bar{C}(3,3) = C(3,3)/\rho g L^2 \\ C(3,4) = \rho g \iint_{S_b} y n_3 dS & \bar{C}(3,4) = C(3,4)/\rho g L^3 \\ C(3,5) = -\rho g \iint_{S_b} x n_3 dS & \bar{C}(3,5) = C(3,5)/\rho g L^3 \\ C(4,4) = \rho g \iint_{S_b} y^2 n_3 dS + \rho g \forall z_b - m g z_g & \bar{C}(4,4) = C(4,4)/\rho g L^4 \\ C(4,5) = -\rho g \iint_{S_b} x y n_3 dS & \bar{C}(4,5) = C(4,5)/\rho g L^4 \\ C(4,6) = -\rho g \forall x_b + m g x_g & \bar{C}(4,6) = C(4,6)/\rho g L^4 \\ C(5,5) = \rho g \iint_{S_b} x^2 n_3 dS + \rho g \forall z_b - m g z_g & \bar{C}(5,5) = C(5,5)/\rho g L^4 \\ C(5,6) = -\rho g \forall y_b + m g y_g & \bar{C}(5,6) = C(5,6)/\rho g L^4 \end{array}$$

where  $C(i,j) = C(j,i)$  for all  $i,j$ , except for  $C(4,6)$  and  $C(5,6)$ . For all other values of the indices  $i,j$ ,  $C(i,j) = 0$ . In particular,  $C(6,4) = C(6,5) = 0$ .

In  $C(4,4)$ ,  $C(4,6)$ ,  $C(5,5)$  and  $C(5,6)$ ,  $m$  denotes the body mass. When Alternative form 1 is used for the FRC file (Section 3.3) the body mass is computed from the relation  $m = \rho \forall$ . When Alternative form 2 is used for the FRC file (Section 3.4) the body mass is defined by EXMASS(3,3).

## 4.2 ADDED-MASS AND DAMPING COEFFICIENTS

$$A_{ij} - \frac{i}{\omega} B_{ij} = \rho \iint_{S_b} n_i \varphi_j dS$$

$$\bar{A}_{ij} = \frac{A_{ij}}{\rho L^k} \quad \bar{B}_{ij} = \frac{B_{ij}}{\rho L^k \omega}.$$

where  $k = 3$  for  $i, j = 1, 2, 3$ ,  $k = 4$  for  $i = 1, 2, 3$ ,  $j = 4, 5, 6$  or  $i = 4, 5, 6$ ,  $j = 1, 2, 3$  and  $k = 5$  for  $i, j = 4, 5, 6$ .

## 4.3 EXCITING FORCES

a) Exciting forces from the Haskind relations

$$X_i = -i\omega\rho \iint_{S_b} \left( n_i \varphi_0 - \varphi_i \frac{\partial \varphi_0}{\partial n} \right) dS$$

b) Exciting forces from direct integration of hydrodynamic pressure

$$X_i = -i\omega\rho \iint_{S_b} n_i \varphi_D dS$$

$$\bar{X}_i = \frac{X_i}{\rho g A L^m},$$

where  $m = 2$  for  $i = 1, 2, 3$  and  $m = 3$  for  $i = 4, 5, 6$ .

## 4.4 BODY MOTIONS IN WAVES

Two alternative procedures are followed to evaluate the body motions in waves, corresponding respectively to the Alternative 1 (Section 3.3) and Alternative 2 (Section 3.4) FRC control files.

In Alternative 1, which is restricted to a body in free stable flotation without external constraints, the following relations hold

$$m = \rho \nabla$$

$$x_b = x_g, \quad y_b = y_g$$

where  $m$  is the body mass and  $(x_g, y_g, z_g)$  are the coordinates of the center of gravity.

The inertia matrix is defined as follows.

$$M = \begin{pmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_{11} & I_{12} & I_{13} \\ mz_g & 0 & -mx_g & I_{21} & I_{22} & I_{23} \\ -my_g & mx_g & 0 & I_{31} & I_{32} & I_{33} \end{pmatrix}. \quad (4.1)$$

WAMIT equates the body mass to the mass of the displaced water in free flotation. The moments of inertia  $I_{ij}$  are defined in terms of the corresponding radii of gyration  $r_{ij}$ , defined by the relation

$$I_{ij} = \rho \forall r_{ij} |r_{ij}|.$$

The array XPRDCT(I,J) input to WAMIT contains the radii of gyration input with the same units of length as the length scale ULEN defined in the panel data file.

In the Alternative 2 format of the FRC file the matrices  $M_{ij} + M_{ij}^E$ ,  $B_{ij}^E$  and  $C_{ij}^E$  are input by the user to include the possibility of external force/moment constraints acting on the body.

The complex amplitudes of the body's motions  $\xi_j$  are obtained from the solution of the  $6 \times 6$  linear system, obtained by applying Newton's law

$$\sum_{j=1}^6 \left[ -\omega^2 (M_{ij} + M_{ij}^E + A_{ij}) + i\omega (B_{ij} + B_{ij}^E) + (C_{ij} + C_{ij}^E) \right] \xi_j = X_i.$$

where the matrices  $M_{ij}^E$ ,  $B_{ij}^E$  and  $C_{ij}^E$  are included only in the Alternative 2 case. Note that in the Alternative 2 case the user must specify the body inertia matrix  $M_{ij}$  and include it in the total inertia matrix  $M_{ij} + M_{ij}^E$  specified in the FRC file.

The non-dimensional definitions of the body motions are

$$\bar{\xi}_i = \frac{\xi_i}{A/L^n},$$

where  $n = 0$  for  $i = 1, 2, 3$  and  $n = 1$  for  $i = 4, 5, 6$ . The rotational motions ( $\xi_4, \xi_5, \xi_6$ ) are measured in radians.

## 4.5 HYDRODYNAMIC PRESSURE

The complex unsteady hydrodynamic pressure on the body boundary or in the fluid domain is related to the velocity potential by the linearized Bernoulli equation

$$p = -\rho \frac{\partial \varphi}{\partial t}.$$

The total velocity potential is defined by

$$\varphi = \varphi_D + i\omega \sum_{j=1}^6 \xi_j \varphi_j,$$

where the radiation and diffraction velocity potentials are defined in Section 12.1. In order to render the velocity potential and the hydrodynamic pressure non-dimensional, we define

$$\bar{p} = \frac{p}{\rho g A} = \bar{\varphi}_D + KL \sum_{j=1}^6 \bar{\xi}_j \bar{\varphi}_j.$$

where  $K = \omega^2/g$  and

$$\bar{\varphi}_D = \frac{\varphi_D}{igA/\omega}, \quad \bar{\varphi}_j = \frac{\varphi_j}{L^{n+1}}$$

with  $n = 0$  for  $j = 1, 2, 3$  and  $n = 1$  for  $i = 4, 5, 6$ .

The body pressure can be evaluated separately for the diffraction or radiation problems by following the procedure described in Section 4.12. When the radiation components are output separately, the nondimensional pressure due to  $j$ th mode is defined by

$$\bar{p} = \frac{p}{\rho g \xi_j L^n} = KL \bar{\varphi}_j \quad \text{or} \quad \bar{p} = \frac{p}{\rho a_j L^{n+1}} = -\bar{\varphi}_j$$

where  $a_j$  is the acceleration in the same unit as the gravitational constant specified in the GDF file for  $j = 1, 2, 3$  and the angular acceleration in  $rad/s^2$  for  $j = 4, 5, 6$ .  $n = 0$  for  $j = 1, 2, 3$  and  $n = 1$  for  $j = 4, 5, 6$ .

## 4.6 FREE-SURFACE ELEVATION

The free surface elevation is obtained from the dynamic free-surface condition

$$\eta = -\frac{1}{g} \left( \frac{\partial \varphi}{\partial t} \right)_{z=0},$$

and in non-dimensional form

$$\bar{\eta} = \frac{\eta}{A} = \left( \bar{\varphi}_D + KL \sum_{j=1}^6 \bar{\xi}_j \bar{\varphi}_j \right)_{z=0},$$

where  $\bar{\varphi}$  is defined as in Section 4.5. [Note that the non-dimensional field hydrodynamic pressure and wave elevation are equal to the non-dimensional velocity potential at the respective positions.]

These parameters can be evaluated separately for the diffraction or radiation problems by following the procedure described in Section 4.10. When the radiation components are output separately, the nondimensional free-surface elevation due to  $j$ th mode is defined by

$$\bar{\eta} = \frac{\eta}{\xi_j L^n} = KL \bar{\varphi}_j$$

where  $n = 0$  for  $j = 1, 2, 3$  and  $n = 1$  for  $j = 4, 5, 6$ .

The evaluation of the pressure or free-surface elevation requires special caution close to the body surface. Within a distance on the order of the dimensions of the adjacent panel(s), field-point quantities cannot be computed reliably. More specific limits can be ascertained by performing a sequence of computations and studying the continuity of the result. Approaching the body along a line normal to the centroid of a panel will minimize this problem. See Reference [12] regarding the computation of run-up at the intersection of the body and free surface.

## 4.7 VELOCITY VECTOR ON THE BODY AND IN THE FLUID DOMAIN

The non-dimensional velocities evaluated by WAMIT are defined in vector form by

$$\bar{\mathbf{V}} = \frac{\mathbf{V}}{igA/(\omega L)} = \bar{\nabla} \bar{\varphi}_D + KL \sum_{j=1}^6 \bar{\xi}_j \bar{\nabla} \bar{\varphi}_j$$

where

$$\bar{\nabla} = L \nabla$$

is the non-dimensional gradient operator.

These parameters can be evaluated separately for the diffraction or radiation problems by following the procedure described in Section 4.10.

The evaluation of the velocity requires special caution close to the body surface, in the same manner as the pressure and free-surface elevation. (See Section 4.6 above.) When the radiation components are output separately, the nondimensional velocity due to  $j$ th mode is defined by

$$\bar{V} = \frac{V}{v_j L^n} = \bar{\nabla} \bar{\varphi}_j$$

where  $v_j$  denotes the velocity of the body for  $j = 1, 2, 3$  and the angular velocity for  $j = 4, 5, 6$ .  $n = 0$  for  $j = 1, 2, 3$  and  $n = 1$  for  $j = 4, 5, 6$ .

## 4.8 MEAN DRIFT FORCE AND MOMENT

The definition of the non-dimensional mean drift force and moment in unidirectional waves is

$$\bar{F}_i = \frac{F_i}{\rho g A^2 L^k}$$

where  $k = 1$  for the forces ( $i = 1, 2, 3$ ), and  $k = 2$  for the moments ( $i = 4, 5, 6$ ).

For bi-directional waves of the same period, with complex amplitudes ( $A_1, A_2$ ) and corresponding angles of incidence ( $\beta_1, \beta_2$ ), the nondimensional outputs  $\bar{F}_i(\beta_1, \beta_2)$  are the coefficients such that the total dimensional mean drift force or moment exerted on the body is given by the equation

$$F_i(\beta_1, \beta_2) = \rho g L^k (|A_1|^2 \bar{F}_i(\beta_1, \beta_1) + |A_2|^2 \bar{F}_i(\beta_2, \beta_2) + 2 \text{Re}[A_1 A_2^* \bar{F}_i(\beta_1, \beta_2)])$$

Note that  $\bar{F}_i^*(\beta_2, \beta_1) = \bar{F}_i(\beta_1, \beta_2)$ , where the asterisk (\*) denotes the complex conjugate.

In Option 8, the evaluation of the horizontal drift force and vertical moment is based on the momentum conservation principle in its general form (see References [4] and [26]), without the assumption of energy conservation. This permits the analysis of cases where the body motions are affected by non-conservative effects, such as external damping. The azimuthal integration required to evaluate the momentum flux is performed by an adaptive quadrature formula in subroutine MDRFTM. The integration is performed iteratively, with convergence specified by the criterion of absolute or relative errors in each drift force less than  $\text{TOL}=10^{-4}$ . The maximum number of iterations is controlled by the parameter MAXMIT. A warning message is displayed in the event that this convergence criterion is not satisfied. See Section 10.1 for further information regarding the interpretation and control of this warning message.

Often the warning message is issued because the length scale parameter ULEN is much smaller than the relevant length scale of the body. Since the drift force increases in proportion to length, and the moment in proportion to  $(\text{length})^2$ , relatively small differences between large values may not be significant. In this case the warning message can be avoided by increasing ULEN to a value more representative of the length.

This force or moment can be either converged for most practical purposes or too small to be important in practice. It is recommended to check the practical importance of this quantity. Further check on the convergence of the result can be made by increasing MAXMIT

gradually. Since the computational time increases exponentially, it is not recommended to use significantly large MAXMIT than the default value.

In Option 9, the evaluation of the drift force and moment is based on integration of the pressure over the body surface, using the relations in [10] and [17], as summarized in Section 12.7. When ICTRSURF=1 the drift force and moment are also evaluated based on the momentum flux across a control surface, using equations (12.57-60).

The mean drift force and moment evaluated from momentum conservation, in Option 8, are defined with respect to the global coordinate system. Conversely, in Options 9 and 9c, the mean drift force and moment evaluated from pressure integration and from momentum flux on a control surface are defined with respect to the body coordinate system.

The mean drift force and moment can be evaluated separately for the diffraction or radiation problems by following the procedure described in Section 4.10.

## 4.9 FORMAT OF NUMERIC OUTPUT FILES

For each of the nine options in the FORCE subprogram, separate output files of numeric data are generated as listed in Section 3.3. The hydrodynamic parameters in these files are output in the same order as in the OUT file, and listed in the following format:

OPTN.1:	PER	I	J	$\bar{A}_{ij}$	$\bar{B}_{ij}$				
OPTN.2:	PER	BETA	I	$\text{Mod}(\bar{X}_i)$	$\text{Pha}(\bar{X}_i)$	$\text{Re}(\bar{X}_i)$	$\text{Im}(\bar{X}_i)$		
OPTN.3:	PER	BETA	I	$\text{Mod}(\bar{X}_i)$	$\text{Pha}(\bar{X}_i)$	$\text{Re}(\bar{X}_i)$	$\text{Im}(\bar{X}_i)$		
OPTN.4:	PER	BETA	I	$\text{Mod}(\bar{\xi}_i)$	$\text{Pha}(\bar{\xi}_i)$	$\text{Re}(\bar{\xi}_i)$	$\text{Im}(\bar{\xi}_i)$		
OPTN.5P:	PER	BETA	M	K	$\text{Mod}(\bar{p})$	$\text{Pha}(\bar{p})$	$\text{Re}(\bar{p})$	$\text{Im}(\bar{p})$	
OPTN.5VX:	PER	BETA	M	K	$\text{Mod}(\bar{V}_x)$	$\text{Pha}(\bar{V}_x)$	$\text{Re}(\bar{V}_x)$	$\text{Im}(\bar{V}_x)$	
OPTN.5VY:	PER	BETA	M	K	$\text{Mod}(\bar{V}_y)$	$\text{Pha}(\bar{V}_y)$	$\text{Re}(\bar{V}_y)$	$\text{Im}(\bar{V}_y)$	
OPTN.5VZ:	PER	BETA	M	K	$\text{Mod}(\bar{V}_z)$	$\text{Pha}(\bar{V}_z)$	$\text{Re}(\bar{V}_z)$	$\text{Im}(\bar{V}_z)$	
OPTN.6:	PER	BETA	L		$\text{Mod}(\bar{p})$	$\text{Pha}(\bar{p})$	$\text{Re}(\bar{p})$	$\text{Im}(\bar{p})$	
OPTN.7X:	PER	BETA	L		$\text{Mod}(\bar{V}_x)$	$\text{Pha}(\bar{V}_x)$	$\text{Re}(\bar{V}_x)$	$\text{Im}(\bar{V}_x)$	
OPTN.7Y:	PER	BETA	L		$\text{Mod}(\bar{V}_y)$	$\text{Pha}(\bar{V}_y)$	$\text{Re}(\bar{V}_y)$	$\text{Im}(\bar{V}_y)$	
OPTN.7Z:	PER	BETA	L		$\text{Mod}(\bar{V}_z)$	$\text{Pha}(\bar{V}_z)$	$\text{Re}(\bar{V}_z)$	$\text{Im}(\bar{V}_z)$	
OPTN.8:	PER	BETA <sub>1</sub>	BETA <sub>2</sub>	I	$\text{Mod}(\bar{F}_i)$	$\text{Pha}(\bar{F}_i)$	$\text{Re}(\bar{F}_i)$	$\text{Im}(\bar{F}_i)$	
OPTN.9:	PER	BETA <sub>1</sub>	BETA <sub>2</sub>	I	$\text{Mod}(\bar{F}_i)$	$\text{Pha}(\bar{F}_i)$	$\text{Re}(\bar{F}_i)$	$\text{Im}(\bar{F}_i)$	
		[PER	BETA <sub>1</sub>	BETA <sub>2</sub>	-I	$\text{Mod}(\bar{F}_{io})$	$\text{Pha}(\bar{F}_{io})$	$\text{Re}(\bar{F}_{io})$	$\text{Im}(\bar{F}_{io})$ ]

(Depending on the value of NUMNAM, the filenames OPTN will be replaced by *frc*.)

If option 5 is specified and INUMOPT5=1, as explained in Section 4.12, the numeric output files .5P, .5VX, .5VY, .5VZ contain the separate components of the radiation and diffraction pressure and velocity in the following modified format:

OPTN.5P:	PER	M	K	$\text{Re}(\bar{p}_1)$	$\text{Im}(\bar{p}_1)$	$\text{Re}(\bar{p}_2)$	$\text{Im}(\bar{p}_2)$	...
	PER	BETA	M	K	$\text{Re}(\bar{p}_D)$	$\text{Im}(\bar{p}_D)$		

Here ... denotes the remaining components for modes 3,4,5,6 if the six rigid-body modes are specified for a single body. More generally when different sets of modes are evaluated for one or multiple bodies, these are output in sequence. For each wave period the radiation pressures are listed for all values of M and K before the diffraction pressures. Corresponding formats apply for the fluid velocity components in the files OPTN.5VX, OPTN.5VY, OPTN.5VZ.

- Starting in Version 6.3, the supplementary output file *out.hst* is created in the following format, to output values of the hydrostatic matrix  $C_{ij}$ :

*out.hst*: I J C(I,J)

If Option 5 is specified and IPNLBPT=0, the supplementary output file *gdf.PNL* is created in the following format:

*gdf.PNL*: M K XCT YCT ZCT AREA  $n_x$   $n_y$   $n_z$   $(\mathbf{r} \times \mathbf{n})_x$   $(\mathbf{r} \times \mathbf{n})_y$   $(\mathbf{r} \times \mathbf{n})_z$

If option 6 is specified and INUMOPT6=1 or option 7 is specified and INUMOPT7=1, the numeric output files .6, or .7X, .7Y and 7Z contain the separate components of the

radiation and diffraction pressure and velocity in the following modified format:

```
OPTN.6: PER L      Re( $\bar{p}_1$ ) Im( $\bar{p}_1$ ) Re( $\bar{p}_2$ ) Im( $\bar{p}_2$ ) ...
        PER BETA L      Re( $\bar{p}_D$ ) Im( $\bar{p}_D$ )
```

Here ... denotes the remaining components for modes 3,4,5,6 if the six rigid-body modes are specified for a single body. More generally when different sets of modes are evaluated for one or multiple bodies, these are output in sequence. For each wave period the radiation pressures are listed for all values of L before the diffraction pressures. Corresponding formats apply for the fluid velocity components in the files OPTN.7X, OPTN.7Y, OPTN.7Z. (See section 4.12.)

If Option 6 or 7 is specified, the supplementary output file *frc.FPT* will be created in the following format:

```
OPTN.FPT: L XFIELD(L) YFIELD(L) ZFIELD(L)
```

Except as noted below, the definitions of parameters in these files are as follows:

I, J: Mode indices

M: Index for quadrant (2 planes of symmetry) or half (1 plane of symmetry).

(If no planes of symmetry are specified, or if IPNLBPT>0, then M=1.)

K: Index for panels on the body surface

L: Index for field points

PER: Period

BETA: Wave heading

BETA<sub>1</sub>, BETA<sub>2</sub>: Two wave headings for the mean drift forces and moments

XCT, YCT, ZCT: Dimensional global coordinates of panel centroid.

AREA: Dimensional value of the area of a panel

$n_x, n_y, n_z$ : Components of the unit vector normal to *K*-th panel in local coordinate system

$(\mathbf{r} \times \mathbf{n})_x, (\mathbf{r} \times \mathbf{n})_y, (\mathbf{r} \times \mathbf{n})_z$ : Components of the cross product of the position vector to the centroid of the *K*-th panel and its normal vector, in the local coordinate system. Here  $\mathbf{r}$  is given in dimensional units.

XFIELD, YFIELD, ZFIELD: Dimensional global coordinates of the field point

All output quantities are nondimensionalized as defined in Sections 4.2-8. Complex quantities are defined by the magnitude (Mod), phase in degrees (Pha), and also in terms of the real (Re) and imaginary (Im) components. The phase is relative to the phase of an incident wave at the origin of the global coordinates system.

In Option 5, when IPNLBPT  $\neq$  0, the index M refers to the body index and K refers to the body point in the order listed in the .bpi input file and .bpo output file. The file *gdf.pnl* is only output when IOPTN(5)>0 and IPNLBPT=0. If ILOWHI=1 the data output in this file differ from those shown above as follows:

K: Index for points on the body surface (See section 4.10)

XCT, YCT, ZCT: Dimensional global coordinates of points

AREA: Product  $J\delta U\delta V$  where  $J$  is the Jacobian at the point, and  $\delta U$ ,  $\delta V$  denote the differential increments between points in parametric coordinates.

$n_x, n_y, n_z$ : Components of the unit vector normal to the body surface at each point

$(\mathbf{r} \times \mathbf{n})_x, (\mathbf{r} \times \mathbf{n})_y, (\mathbf{r} \times \mathbf{n})_z$  : Components of the cross product of the position vector at each point.

In Option 8, the mean force and moment are output only for modes I=1, 2 and 6, corresponding to the two horizontal forces and yaw moment, respectively.

In Option 9, the six components of the mean forces and moments,  $\bar{F}_i$ , are output on the first six lines, with positive indices (i=1,2,...,6). These are the components of the force and moment vectors, defined with respect to the inertial reference frame corresponding to the mean position of the body coordinate system. When **IRAD**  $\neq$  -1, three additional components of the moment  $\bar{F}_{i_o}$ , are output and identified by negative indices (i=-4,-5,-6). These are the components of the moment about the moving origin, denoted by 'o' in Figure 12-2. In all cases the components of the vector force and moment are defined with respect to the inertial (mean) coordinate system.

In Option 9, if ICTRSUFT=1, the drift forces evaluated from momentum flux on the control surface are output in the numeric output file OPTN.9c, in the same format as shown above for OPTN.9.

If NBODY>1, the panels of all bodies are merged with a common index K, following the same order as the body numbers in the global GDF file (See Chapter 7).

## 4.10 BODY PRESSURE OUTPUT FOR THE HIGHER-ORDER METHOD

If the higher-order method is used (ILOWHI=1), Option 5 is selected in the FRC file, and IPNLBPT=0, the pressure and the fluid velocity on the body surface are output at the points corresponding to equally spaced points in parametric space. These points are defined in parametric space as the midpoints of the set of  $(KU + 1) * (KV + 1)$  panel subdivisions on each patch (see Chapter 6). The coordinates, the extended normal vector corresponding to 6 rigid body modes and the Jacobian are output in the .pnl file. The value of the Jacobian at the prescribed point replaces the panel area in the format shown in Section 4.9. The pressure and the fluid velocity vector at these points are output in the files 5p, 5vx, 5vy and 5vz in the same format as shown in Section 4.9. ( If IPNLBPT≠0 is assigned in the configuration file an alternative option is utilized with the points on the body surface specified by the user, as described in Section 4.11.)

When the above options are specified a second output file .5pb is also generated. This file contains the B-spline coefficients and other relevant parameters for the evaluation of the pressure and its derivatives on the body surface. The total pressure coefficient ( $\varphi$ ), the diffraction pressure ( $\varphi_D$ ) and the radiation pressure ( $\varphi_R$ ) are output separately. The radiation pressure has as many components as the number of modes specified in the POT file, including generalized modes. Following the definition of the nondimensional pressure (Section 4.5) these three components are related by the equation

$$\varphi = \varphi_D + KL \sum_j \xi_j \varphi_j$$

where  $KL$  is the nondimensional infinite depth wavenumber,  $\xi_j$  is the nondimensional motion amplitude and  $j$  is the mode index.

*The total pressure coefficient is output always. The diffraction pressure coefficient is output when IRAD> -1 and IDIFF>-1. Since the total pressure is the same as the diffraction pressure if IRAD=-1, the diffraction pressure is not output in this case. The radiation pressure coefficient is output when IRAD> -1.*

The data in the .5pb file is useful for special post-processing purposes, such as for interfacing with structural loads analyses. The content of the .5pb numeric output file is listed below:

```
HEADER
ISX, ISY
ULEN
NPATCH
IRAD, IDIFF
NPER, NBETA
NEQN
NLHS
NDFR
NBODY
```

```

((XBODY(L, J), L=1, 4), J=1, NBODY)
((XBCS(L, J), L=1, 2), J=1, NBODY)
(IBPTH(L), L=1, NPATCH)
(IBMOD(L), L=1, NBODY)
(IGEO(J), J=1, 8)
(ILHS(J), J=1, 4)
(IFLAT(L), L=1, NPATCH)
(KU(L), KV(L), NU(L), NV(L), L=1, NPATCH)
(NMDS(J), J=1, 4)
(ICOL(J), J=1, NDFR)
((MDS(L, J), L=1, NDFR), J=1, 4)
(BETA(NB), NB=1, NBETA) (omit if IDIFF=-1)
Loop over number of periods (repeat NPER times)
  PER(IP), WVNFIN(IP), WVNUM, IFREQ
  IF block starts (if IFREQ=0)
    IF block starts (if IRAD>-1 and IDIFF>-1)
      Loop over wave-headings starts (repeat NBETA times)
        (WRAO(IM, NB), IM=1, NDFR)
      Loop over wave-headings ends (repeat NBETA times)
      Loop over wave-headings starts(repeat NBETA times)
        Loop over number of symmetric images (repeat MXNLHS times)
          Loop over number of patches (repeat NPATCH times)
            (WPRS(I, M, NB), I=NP+1, NQ) (omit if IFLAT(L)=-1)
          End of the loop over number of patches
        End of the loop over symmetric images
      Loop over wave-headings ends (repeat NBETA times)
    IF block ends (if IRAD>-1 and IDIFF>-1)
  IF block starts (if IDIFF >-1)
    Loop over wave-headings starts (NBETA times)
      Loop over number of symmetric images (repeat MXNLHS times)
        Loop over number of patches (repeat NPATCH times)
          (WBD(I, M, NB), I=NP+1, NQ) (omit if IFLAT(L)=-1)
        End of the loop over number of patches
      End of the loop over symmetric images
    Loop over wave-headings ends (NBETA times)
  IF block ends (if IDIFF >-1)
  IF block ends (if IFREQ=0)
  IF block starts ( if IRAD>-1 )
    Loop over left hand side starts (repeat NLHS times)
      Loop over number of modes for each left-hand-side
      MDI (mode index)
      Loop over number of symmetric images (repeat MXNLHS times)
        Loop over number of patches (repeat NPATCH times)
          (WBR(I, ICOL(MDI)), I=NP+1, NQ) (omit if IFLAT(L)=-1)

```

End of the loop over number of patches  
 End of the loop over symmetric images  
 End of the loop over number of modes for each left-hand-side  
 Loop over left hand side ends (repeat NLHS times)  
 IF block ends ( if IRAD>-1 )  
 End of the loop over number of periods  
 NP+1 and NQ are the pointers of the first and the last B-spline coefficients of the unknown velocity potential on patch L.  
**HLINE**: header line  
**ISX,ISY**: Symmetry index (1/0 = symmetric/asymmetric).  
**ULEN**: Characteristic length specified in GDF.  
**NPATCH**: Number of patches.  
**IRAD, IDIFF**: Radiation/diffraction problem indices.  
**NPER, NBETA**: Number of periods and wave headings.  
**NEQN** : The total number of unknown B-spline coefficients.  
**NLHS** : Number of components to be solved when the total solution is decomposed into symmetry and antisymmetry components for the body having geometric symmetry  
**NDFR** : The total number of degrees of freedom. It equal to the sum of the degrees of freedom of each body.  
**NBODY** : Total number of bodies.  
**XBODY** : Normalized coordinates of the origin of body coordinate system and its orientation relative to the global coordinates system.  
**XBCS** : XBCS(1,I) and XBCS(2,I) are cosine and sine of XBODY(4,I)  
**IBPTH(L)** : Body index for patch index L.  
**IBMOD(N)** : Global modes counter. Number of modes prior to the present body N.  
**IGEO** : Parameter used to determine the sign of the pressure/velocity on the reflected patches (see MODE.F)  
**ILHS** : Pointer of the given LHS among NLHS components  
**IFLAT** : Index for patches on the free surface.(IFLAT=-1, patches on interior free surface. IFLAT=1, patches for flat physical surface on the free surface. IFLAT=0, patches not on the free surface.)  
**KU,KV,NU,NV**: Orders and panels  
**NMDS** : For given LHS, total number of modes of radiation problem.  
**MDS** : For given LHS, MDS stores NMDS modes indices.  
**ICOL** : The solution such as motion amplitude is stored in the order which is not ascending from mode 1 (surge). ICOL stores the pointer in that sequence for all modes.  
**BETA** : Wave headings  
**PER WVNFIN WVNINF IFREQ**: Period, finite depth wave number, infinite depth

wave number, period index (IFREQ=0: normal period, IFREQ=1: infinite or zero period).  
When IFREQ=1, the total and diffraction pressure coefficient are not output in .5pb.

**WRAO (I,J):** Complex motion amplitude (I: modes, J:wave heading)

**WPRS:** Total pressure coefficient (I: unknown coefficient, M:reflection J:wave heading)

**WBD:** Diffraction pressure coefficient (I: unknown coefficient, M:reflection J:wave heading)

**MDI:** Mode index.

**WBR:** Radiation pressure coefficient (I: unknown coefficient, ICOL(MDI):pointer of mode MDI)

## 4.11 BODY PRESSURE AND FLUID VELOCITY AT SPECIFIED POINTS

If  $IOPTN(5) > 0$ , the hydrodynamic pressure and fluid velocity on the body surface can be evaluated. The points where the pressure is evaluated depend on the parameter  $IPNLBPT$  in the configuration file. In the default case  $IPNLBPT=0$ , the pressure is evaluated at the panel centroids in the low-order method ( $ILOWHI=0$ ) or at a set of uniformly spaced parametric points on each patch in the higher-order method ( $ILOWHI=1$ ).

If  $IPNLBPT \neq 0$  the body pressure is evaluated at points on the body which are specified by the user in a special input file *gdf.bpi* (Body Point Input). The format of this file is as follows:

```
header
NBPT
X(1) Y(1) Z(1)
X(2) Y(2) Z(2)
.
.
X(NBPT) Y(NBPT) Z(NBPT)
```

Note that the filename of this file must be the same as the filename of the .GDF file.

If  $IPNLBPT > 0$ , the data in the .bpi file is read and interpreted to be in dimensional body-fixed coordinates. If  $IPNLBPT < 0$ , the data in the .bpi file is read and interpreted to be in dimensional global coordinates. (The relationship between these two coordinate systems is defined by the array  $XBODY$ , as defined in Section 3.1.)

The procedure followed to evaluate the body pressure at these specified points is different in the low-order ( $ILOWHI=0$ ) and higher-order ( $ILOWHI=1$ ) solutions. These are described separately below.

If  $ILOWHI=0$ , the solution is based on piecewise constant values of the potential on each panel based on collocation at the panel centroids. In order to evaluate the pressure at other points an interpolation procedure is adopted. This interpolation is based on a user-specified number  $NNEAR$  of nearest panel centroids. The parameter  $NNEAR$  is determined from the absolute value of the input parameter  $IPNLBPT$ .  $IPNLBPT=4$  is recommended, when the input points are in body-fixed coordinates. In this case the program searches and identifies the four nearest panel centroids to each specified input point, and assigns weights to each of these panels based on the (inverse) distance to each centroid. The pressure is output in the .5p numeric output file with the following format:

```
OPTN.5P: PER BETA IBODY IPOINT Mod( $\bar{p}$ ) Pha( $\bar{p}$ ) Re( $\bar{p}$ ) Im( $\bar{p}$ )
```

This format and the definitions of the data are the same as in Section 4.9, except that the index  $IBODY$  is used to specify the body index and  $IPOINT$  is used to specify the index of the input point in the .bpi file ( $J=1,2,\dots,NBPT$ ) for each body. Similar output files .5vx,

.5vy, .5vz contain the components of the fluid velocity on the body surface in the same format, when IOPTN(5) $\geq$ 2. In addition to these hydrodynamic outputs, a supplementary output file *gdf.bpo* (Body Point Output) is created with the following format:

*gdf.BPO*: M N1 R1 N2 R2 N3 R3 N4 R4

Here M is the quadrant index, N1 is the panel index of the nearest panel and R1 is the radial distance from the specified point  $(x, y, z)$  to the centroid of the panel. Successive pairs (Ni, Ri) are the index and radial distance to the other panel centroids, where (i=1,2,...,NNEAR). (In the example shown above NNEAR=4.)

In the higher-order method (ILOWHI=1) the solution for the velocity potential and pressure is represented by continuous B-splines on each patch. For each specified input point  $(x, y, z)$  the program searches for the patch index and  $(U, V)$  coordinates of the point on this patch which is closest to the input point. The pressure is evaluated at the corresponding point  $(U, V)$  and output in the .5p and .5v\* files with the same format as shown above. In this case the supplementary output file *gdf.bpo* contains the following data for each point:

*gdf.BPO*: K M NP U V R I XI XN

Here K is the body point index, M is the quadrant index, NP is the patch index, and  $(U, V)$  are the parametric coordinates on the patch. R is the radial distance from the point  $(U, V)$  on the patch to the specified  $(x, y, z)$  point. An iterative procedure is used to find  $(U, V)$ , with a specified convergence tolerance of 1.0E-4 for the radial distance in nondimensional Cartesian coordinates. (When the length scale of the patch is larger than 1.0 the tolerance is increased by a factor equal to this length scale, estimated from the Jacobian of the parametric transformation at the center of the patch.) I is the number of iterations. A maximum of 16 iterations are used in this search, and if I=17 this indicates nonconvergence of the search. A warning message is generated if nonconvergence occurs for one or more input points, showing the total number of unconverged points. XI is the position vector of the output point on the body surface and XN is the normal vector at XI, both in body coordinates system.

In both the low-order and higher-order implementations, the input data in the .bpi file should correspond to points which lie as close as possible to the body surface.

If points in the BPI files are very close to intersections of adjacent patches in the higher-order method, the index NP in the BPO file should be checked to verify that the correct patch is used, especially in cases where there is ambiguity between the pressure on a conventional patch and the pressure jump on a dipole patch. Similarly, in the low-order method, the panel indices, N1, N2, ... in the BPO file can be checked to verify the corresponding output at the points in BPI files is the pressure obtained from those on the conventional body panels or the pressure jump on the dipole panels.

## 4.12 RADIATION AND DIFFRACTION COMPONENTS OF THE PRESSURE AND VELOCITY

If  $\text{IOPTN}(5) > 0$ , the parameter  $\text{INUMOPT5}$  in the configuration file can be used to control the outputs of the body pressure in the numeric output files. In the default case  $\text{INUMOPT5}=0$  the body pressure output in the numeric output files is the same total pressure as in the formatted .out output file, as defined in Section 4.5. Alternatively, if  $\text{INUMOPT5}=1$ , the separate components  $KL\bar{\varphi}_j$  and  $\bar{\varphi}_D$  are output in the numeric output file .5p and the corresponding components of the fluid velocity on the body surface are output in the files .5vx, .5vy, .5vz. Here  $\bar{\varphi}_j$  is the nondimensional potential in mode  $j$ , and  $\bar{\varphi}_D$  is the nondimensional potential of the diffraction problem with the body fixed, as defined in Section 4.5. In this case the format of the numeric output files is modified, as shown in Section 4.9.

If  $\text{IOPTN}(6) > 0$ , the parameter  $\text{INUMOPT6}$  in the configuration file can be used to control the outputs of the field pressure in the numeric output file. In the default case  $\text{INUMOPT6}=0$ , as in previous versions of WAMIT, the field pressure output in the numeric output files is the same total pressure as in the formatted .out output file, as defined in Section 4.5. Alternatively, if  $\text{INUMOPT6}=1$ , the separate components  $KL\bar{\varphi}_j$  and  $\bar{\varphi}_D$  are output in the numeric output file .6. Similarly, if  $\text{IOPTN}(7) > 0$ , the parameter  $\text{INUMOPT7}$  in the configuration file can be used to control the outputs of the field velocity in the numeric output files, .7x, .7y and .7z. The format of the numeric output files is shown in Section 4.9.

## 4.13 RADIATION PRESSURE AND VELOCITY FOR ZERO AND INFINITE PERIODS

In the two limiting cases of zero and infinite period (or equivalently, infinite and zero frequency), it is possible to evaluate the pressure and velocity on the body (Option 5) and in the fluid (Options 6 and 7). This extension is particularly important in the context of evaluating the corresponding time-domain impulse response functions, as explained in Chapter 13. These extended outputs are only included in the numeric output files if the corresponding parameters  $\text{INUMOPT5}$ ,  $\text{INUMOPT6}$ ,  $\text{INUMOPT7}$  are assigned with value 1 in the .cfg file. The formats of the corresponding numeric output files are explained in Section 4.9.

■ In certain cases it is not possible to evaluate these limits. For a fluid in the tank, the zero frequency limits of the radiation solutions of certain modes cannot be evaluated. Specifically, the radiation solution does not exist, when the fluid volume below the, space-fixed, mean free surface cannot be preserved as in the heave oscillation. Roll and pitch modes do not have the zero frequency limits, unless the tank geometry is symmetric about the center of rotation or the volume happens to be preserved under oscillation. WAMIT outputs the zero frequency limits of all modes but the output associated with these modes should not be interpreted as the limits. They are numerical solutions of ill-posed problems

and should be discarded.

Special definitions are applied to the radiation pressure and velocity in the case of zero frequency, which is identified in the output files by a negative value of the wave period. In general, for nonzero finite values of the frequency, the nondimensional outputs for the radiation pressure and velocity are as defined in Sections 4.5 and 4.7. Thus the output pressure for each radiation mode is  $KL\bar{\varphi}_j$  and the output velocity for each mode is  $KL\bar{\nabla}\bar{\varphi}_j$ . However for the two limiting cases, where  $KL = 0$  or  $KL = \infty$ , the factor  $KL$  is omitted from the outputs for options 5,6,7. The following table summarizes these definitions:

frequency	period	pressure	velocity
$\omega = 0$	PER < 0	$\bar{\varphi}_j$	$\bar{\nabla}\bar{\varphi}_j$
$0 < \omega < \infty$	PER=( $2\pi/\omega$ )	$KL\bar{\varphi}_j$	$KL\bar{\nabla}\bar{\varphi}_j$
$\omega = \infty$	PER=0	$\bar{\varphi}_j$	$\bar{\nabla}\bar{\varphi}_j$