

Report

2007 Annual WAMIT Consortium Meeting

September 25-26, 2007

Woods Hole, Massachusetts

Agenda for 2007 Annual WAMIT Meeting
Room 210, Marine Resource Center, Swope Center, Woods Hole, MA

September 25, 2006:

9:00AM: Welcome

9:10AM: "New features to be introduced in V6.4"
J. N. Newman, WAMIT

9:50AM: "Progress in WAMIT 2nd-order module"
C.-H. Lee, WAMIT

10:30AM: Break

10:50AM: "Progress with trimmed waterlines (using the higher-order method)"
J. N. Newman, WAMIT

11:30AM: "MultiSurf developments for WAMIT"
J. S. Letcher, AeroHydro

12:10PM: Lunch

1:40PM: "ATP Side-by-Side Barge and Ship Experiments at the OTRC"
J.M. Niedzwecki, P. Teigen, R. Mercier and A. Duggal, OTRC

2:20PM: "Second Order Wave Forces on a Semi-Submersible Platform"
J. Sparano, University of San Paulo

3:00 PM: Break

3:30 PM: "Estimating Run-up on a GBS: 1st- and 2nd-Order Contributions from WAMIT" D. Danmeier, Chevron

4:00PM: "Challenges in WAMIT Hydrodynamic Calculations of Offshore Floating Structures" S. Ryu & A. Duggal, SOFEC

5:30PM: Mixer and Dinner

September 26, 2006:

9:00PM: "Wave focussing effects in finite water depth."
P. Teigen, Statoil

9: 40AM: "Evaluation of gravitational moments for generalized modes"
C.-H. Lee, WAMIT

10:10 AM: "Technical discussion"

10:40AM: Break

11:00AM: Business meeting

12:00AM: Lunch

Contents

1. New features to be introduced in V6.4 – J.N. Newman
2. Progress in WAMIT 2nd-order module – C.-H. Lee
3. Progress with trimmed waterlines – J.N. Newman
4. Evaluation of gravitational moments for generalized modes – C.-H. Lee
5. Current Participants
6. Appendices (available at <http://www.wamit.com/publications.htm>)

Numerical studies of directional wavemaker performance, by J.F. Odea and J.N. Newman
(28th American Towing Tank Conference)

Trapping of water waves by moored bodies, by J.N. Newman
(To appear in J. of Engineering Mathematics)

New features to be introduced in V6.4

By J. N. Newman

- Automatic interior free-surface discretization with higher-order method
- Breaking long runs of POTEN
- Walls and Wavemakers
- Trimmed waterlines ([separate talk](#))

Automatic discretization of interior free surface using the higher-order method

(for irregular-frequency removal)

IRR=2: program automatically derives patches for all interior free surfaces

IRR=3: program automatically derives patches for interior free surfaces unless the user has defined these in GDF input (useful for NBODY > 1)

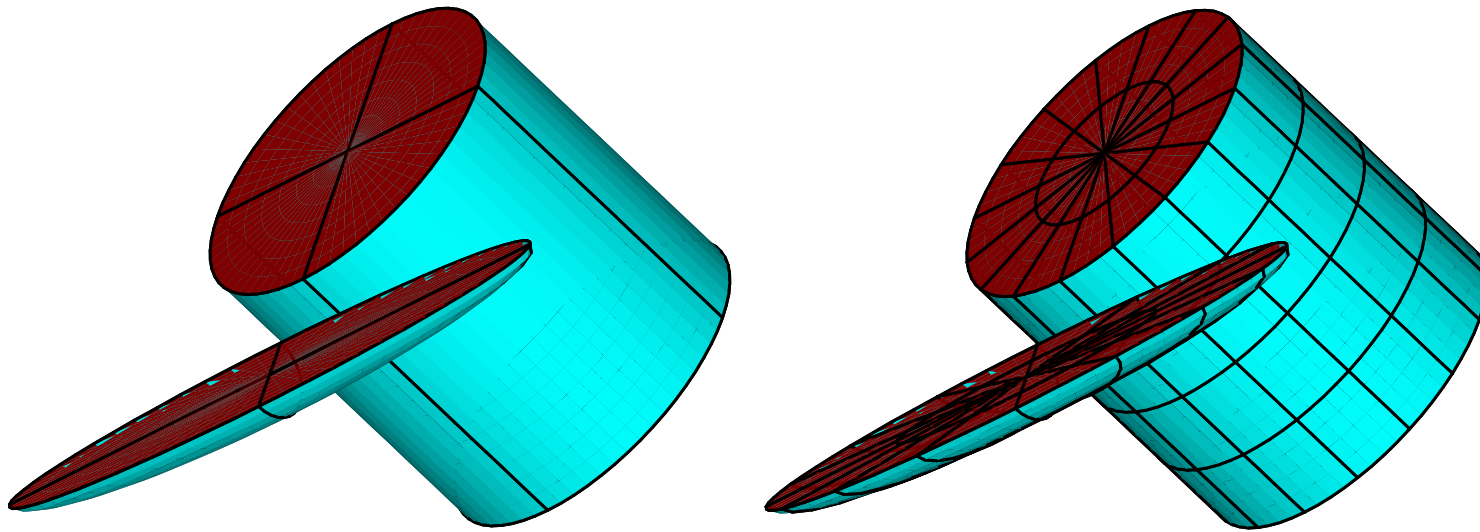
Methodology

- Identify patches and sides which form one or more closed waterlines
- For each closed waterline identify an axis (point or line) in the interior from averaging vertex coordinates
- Define new interior-free-surface patches, one for each exterior patch unless this is not needed (e.g. at a transom stern)
- Use ruled mapping on each new patch
- If waterline slope changes substantially (> 0.5 radian), use polar mapping (e.g. at bow of FPSO or on each waterline of a semi-sub)

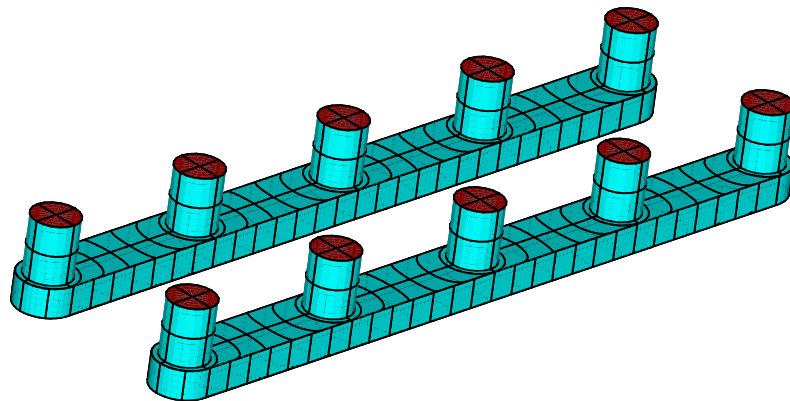
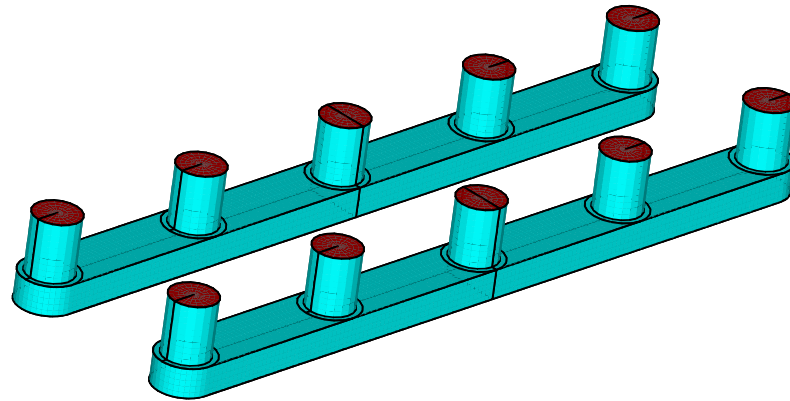
Test 13 (NBODY=2, IRR=3)

Spheroid GDF includes interior free surface

Cylinder GDF does not include interior free surface

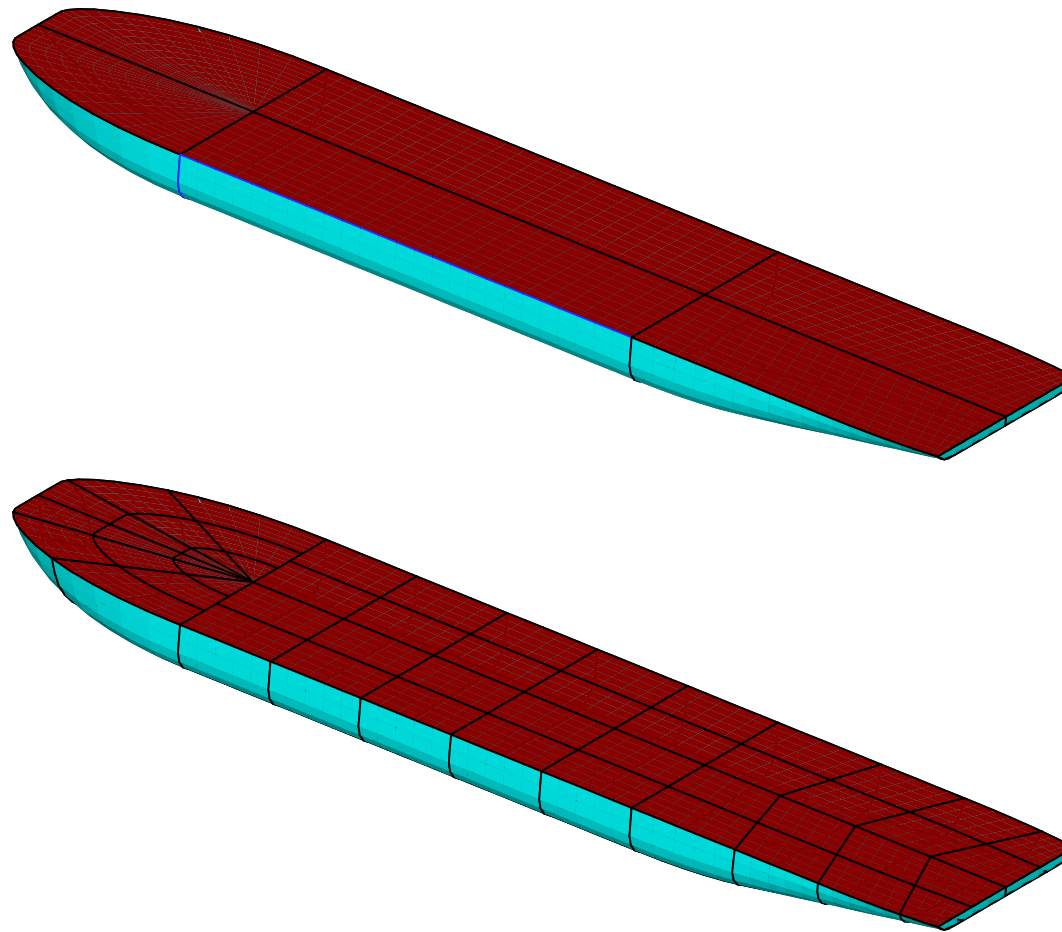


Test 15 (Semi-Sub)

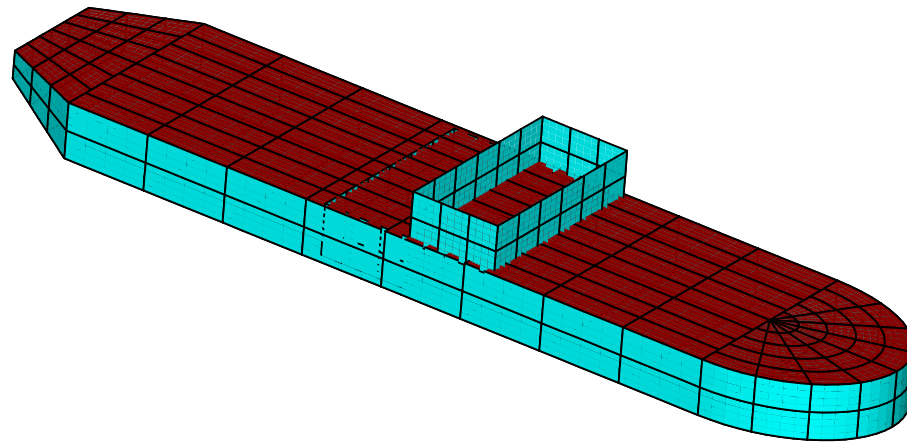
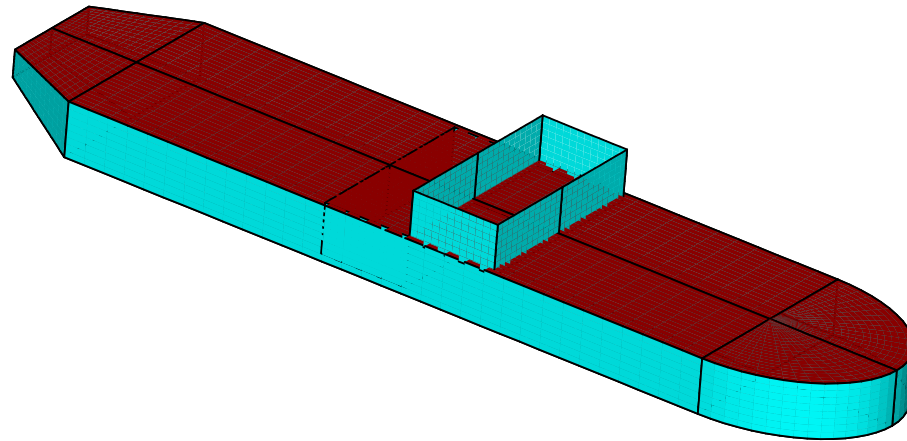


Test 20 (MultiSurf FPSO)

Note polar mapping in bow and no extra patch for transom



Test 22 -- FPSO with two tanks (aft tank is obscured)



Special notes

- Internal free surfaces such as moonpools are not supported (interior free surface must be simply connected)
- Spline parameters for the interior free surface are assigned to give continuity with the exterior patches (same panel subdivision along the waterline and similar panel widths normal to the waterline)
- Algorithms may fail for irregular or complicated cases
- Users should plot the `_pat.dat` and `_pan.dat` output files to verify that the interior free surface representation is OK
- Should be OK to use this option with trimmed waterlines, but this has not yet been tested
- Similar algorithms can be used for the exterior free surface in 2nd-order analysis

Breaking long runs of POTEN

For most first-order WAMIT runs the computational time is spent primarily in POTEN, on the set-up and solution of the linear system of equations for the velocity potential. This occurs in a loop over NPER wave periods, as specified in the .POT input file for the run. It is not unusual to underestimate the time required for the POTEN run. In this circumstance the user may want to break the run and save the solutions which have been computed, for use in FORCE. This has not been possible with V6.3 or prior versions of WAMIT.

Method

A new optional input file can be used, with the reserved filename **break.wam**. If this file does not exist then the run continues normally without breakpoints. If **break.wam** does exist and can be opened, then the program pauses for interactive input by the user at two points within the period loop: (a) before setting up the LHS, and (b) before solving the linear system. Since the **break.wam** file is not read, its contents are irrelevant. This file can be set up either before or during the run. (The simplest procedure is to copy any other existing file to **break.wam**.)

Options for interactive input

If the file **break.wam** exists and can be opened, then at each breakpoint the user is requested to input one of three choices:

B or **b**: Break run and continue with reduced NPER

C or **c**: Continue run and keep BREAK.WAM

D or **d**: Delete BREAK.WAM and continue run

In case **B** the result is the same as if NPER was reduced with the new value $NPER = JPER - 1$, where JPER is the current index of the wave period in the loop.

Walls and Wavemakers

Last year we presented initial work on this topic, restricted to the analysis of the radiated wave field generated by one or two banks of wavemakers situated in the wall(s) $X=0$ and/or $Y=0$ of a semi-infinite wave tank. That capability is included in V6.3. It has been used extensively by John O'Dea to analyze proposed new wavemaker systems for the seakeeping basin at Carderock. (See O'Dea & Newman, ATTC 2007, available for download from www.wamit.com)

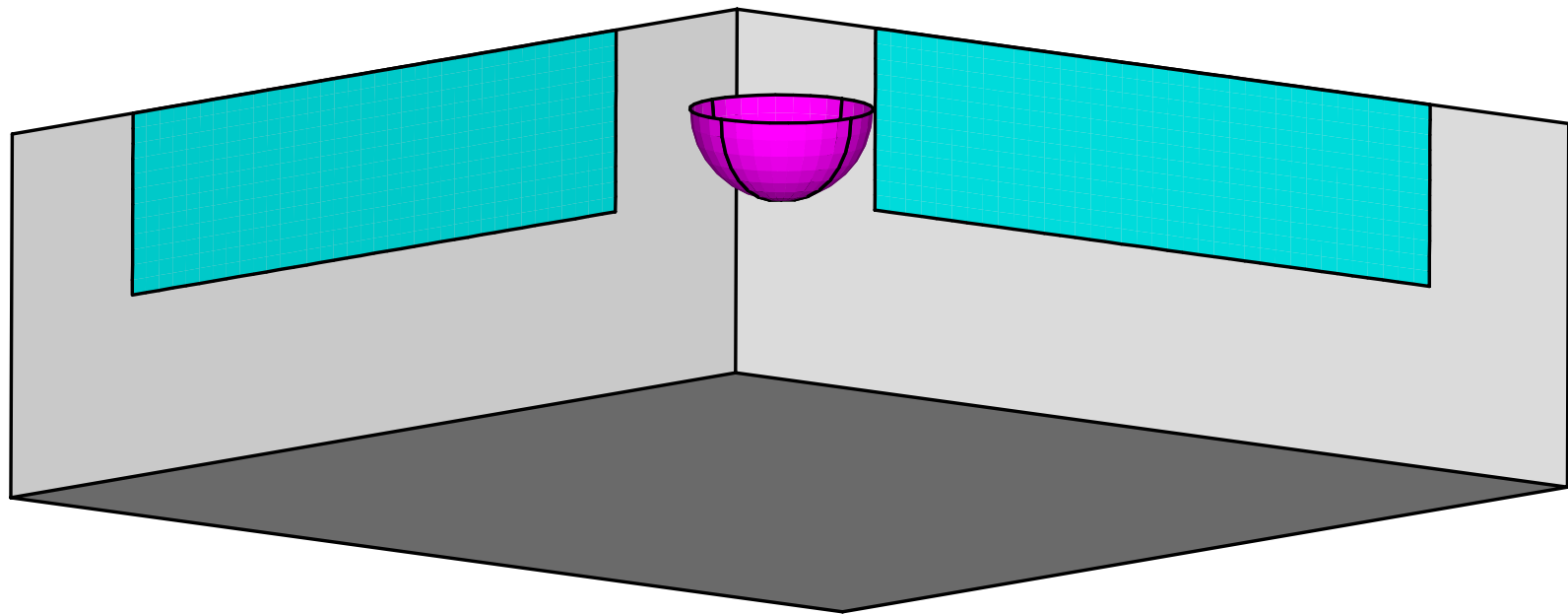
Computational Approach in V6.3

- Represent geometry by low- or higher-order panels/patches (wet side only)
- Set up RHS of linear system (source strength)
- Set velocity potential = 0 on body surface, and skip solution of linear system (ISOLVE=-1)
- Only radiation modes are considered, no incident waves or diffraction.
- Supported outputs include only options 6&7 (wave elevations, pressures, fluid velocities)
- No other bodies can be present in the fluid domain
- Other walls are open boundaries

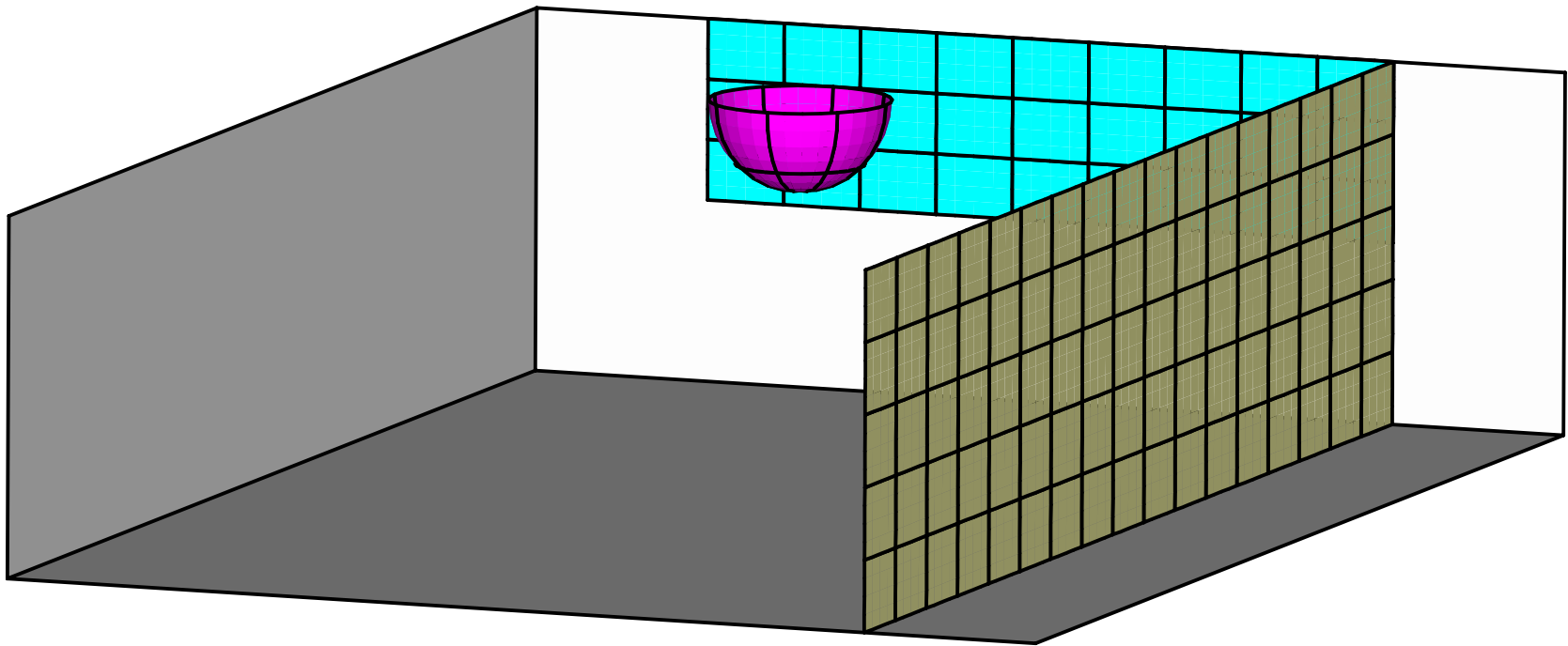
New methodology

- Allow for one or two reflecting walls to be present, coinciding with the plane(s) $X=0$ and/or $Y=0$.
- This approach has always existed in the low-order method (ISX,ISY=-1,-2)
- New approach applies for both low-order and higher-order methods
- All options are supported except momentum drift forces (Option 8)
- Bodies can be present in the fluid
- Wavemakers can be present on the walls

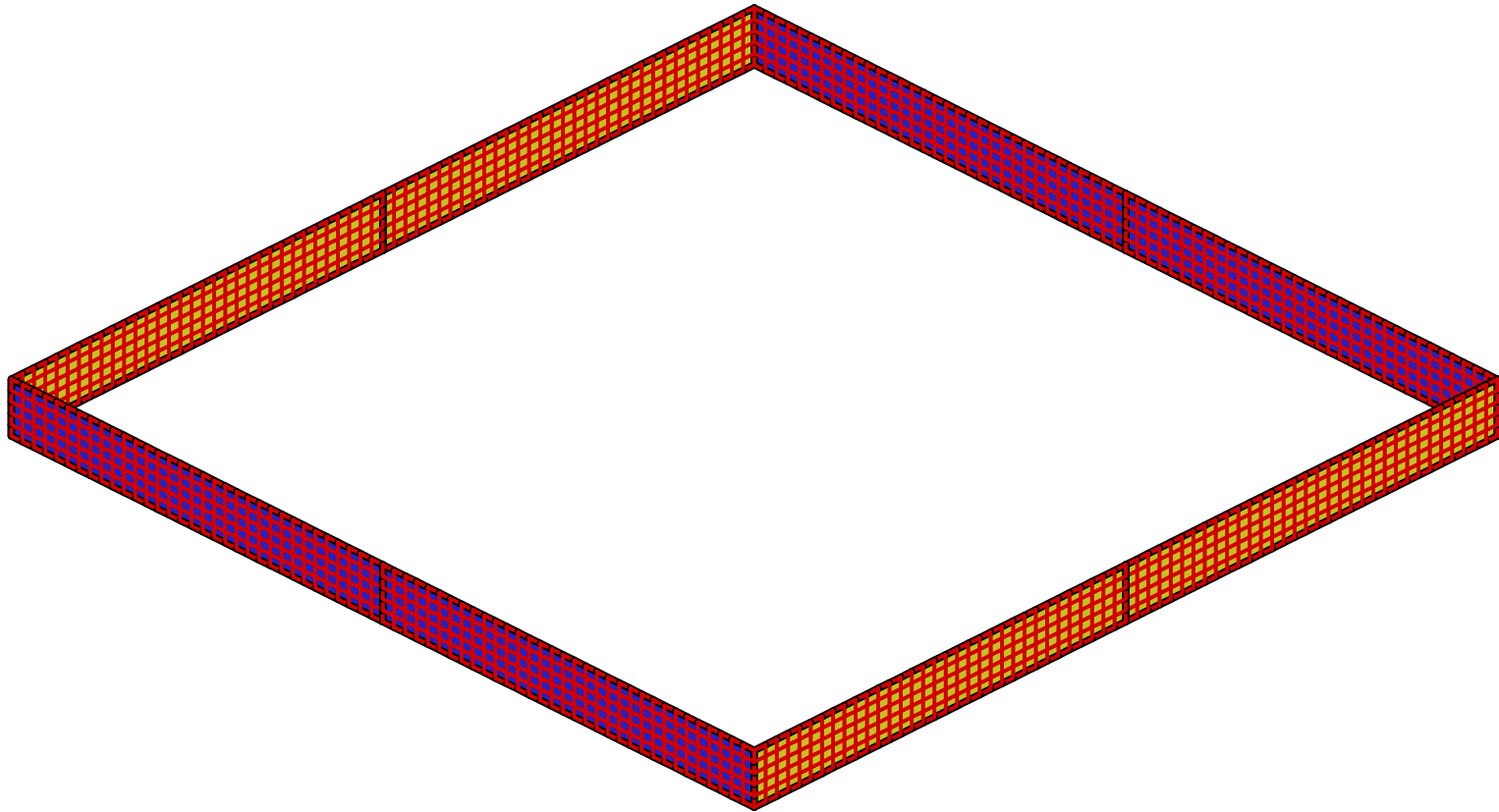
Hemisphere in wave tank with two wavemakers



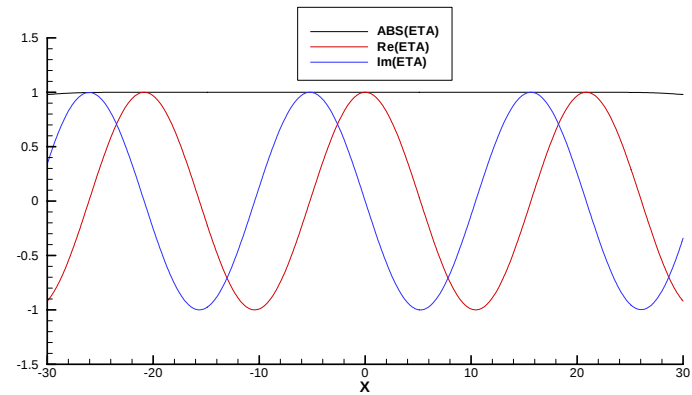
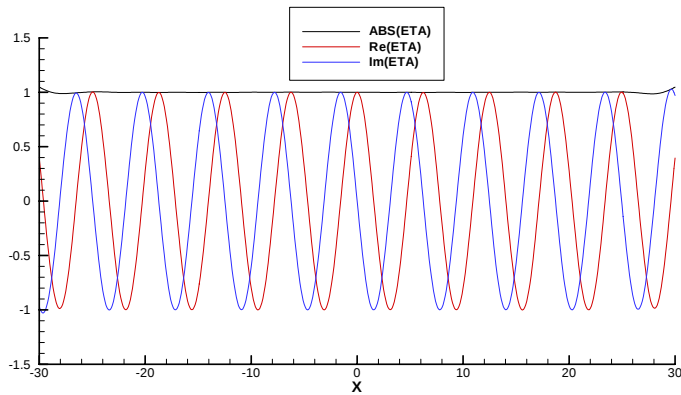
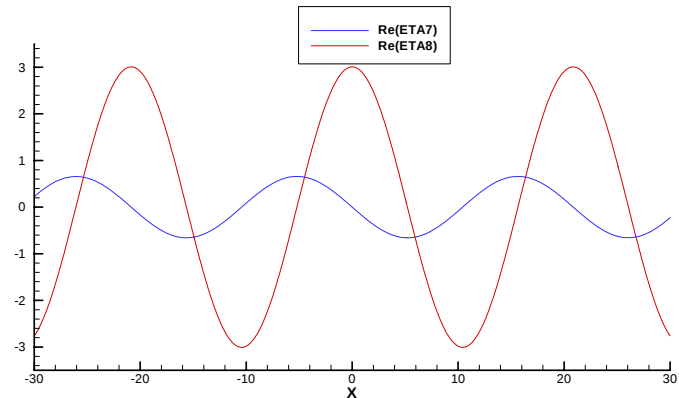
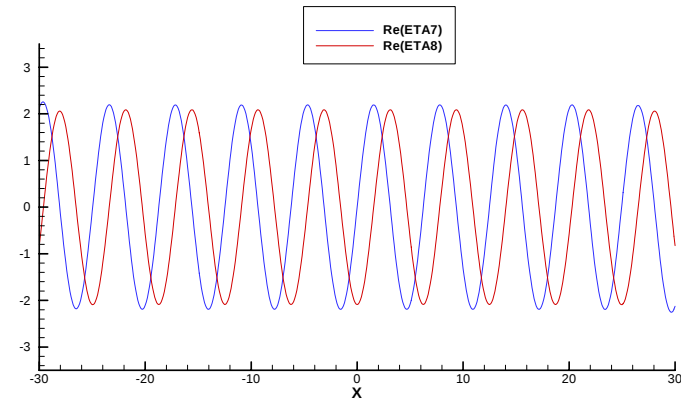
Hemisphere in wave tank with one wavemaker and
a partial wall (dipole patch)



Square wave basin 64x64x4m with fixed sides (yellow) and active ends (blue) -- ISX=ISY=1, NPATCH=2



Waves generated along the tank axis by paddle wavemakers at both ends. The upper figures show the separate modes where the two ends have the same (**symmetric**) and opposite (**antisymmetric**) phases. The lower figures show the combined waves from both modes, combined to produce a single progressive wave with amplitude indicated by the black line. Left figure is 2 second period, right figure 4 second period.



Progress in WAMIT 2nd-order module

- New approach for the Rankine integral of the 2nd-order forcing
- Automatic shifting of the free surface field points

New approach for the Rankine integral of the 2nd-order forcing

Background

- Integration of Rankine source with the 2nd-order forcing based on the quadrature adopted in the higher-order method can be extremely time consuming.

This is because the numbers of the integration node on which the forcing is evaluated can be orders of magnitude more than typical number of panels on the free surface used in low-order method. (The evaluation of forcing on each node requires $O(N)$ effort where N is number of panels/subdivision on the body).

- Because of this difficulty, the 2nd order forcing is obtained in the similar manner as in the low-order method after approximate the body and free surfaces with panels in V6.1S.

Thus the 2nd-order solution may not be as accurate as the linear counterpart. Especially the solution near the waterline can be inaccurate due to the conflict between continuous linear and 2nd-order solutions on the continuous surface (LHS) and the evaluation of the forcing on the discretized panels(RHS).

Background

- Attempt has been made to make integration using the quadrature of the higher order method. In order to avoid repeated evaluation of Rankine influence over the 2nd-order frequency loop, the Rankine influences for all pairs of the source and field points were evaluated and stored as in the low-order method (but orders of magnitude more pairs).

This can reduce the computational cost by a factor of N^2 (by moving the Rankine integral out of 2nd-order frequency loop) but it does not resolve the fundamental problem of large number of integration nodes.

Coding is complicate to store/retrieve the sequences of source points in the successive subdivision for each of the field points and their reflections. The task to identity common nodes to reduce the number of nodes is complicate and time consuming.

- The new approach

- a) The linear pressure and velocity are approximated using the values at the limited number of points on the free surface. Thus the boundary condition at any points on the body and the free surface are evaluated efficiently using the approximation. (Exceptions are the normal velocity on the body due the 2nd-order incident wave and the part of the forcing involving body motion. But these are not as expensive to evaluate as the forcing on the free surface)
- b) The 2nd-order frequency loop is moved as the inner most loop in Rankine integral. The right-hand-side forcing vectors, for all second-order solutions, are set-up at the same time in an efficient manner. The number of solutions can be $NPER * NPER * NBETA * NBETA$. (This is similar to set-up the large number of degrees of freedom for the radiation problem.)

Advantages:

- **Efficient**
- The same quadrature scheme on the body can be easily extended to the free surface integral as well as to line integrals along the waterlines.

Thus the solution procedure is similar to the linear problem and the solution can be as **accurate**

New codes are the logical extension of the existing ones.

Uncertainty:

Accuracy of the approximation of the free surface forcing.

Approximations on the free surface

- Current implementation is based on B-spline approximation. (Effort is made to make the program flexible so that other methods may be tried easily without affecting other parts of the codes significantly.)
- The velocity and the pressure are approximated separately, because the analytic evaluation of the velocity from the potential may be sensitive to the mapping of the free surface patches.
- Quadratic B-splines will be used initially for the approximation and the integration of the forcing (which is in the form of the product of two quadratic B-spline functions) will be based on the 4th order Gauss quadrature.

Relative errors of the pressure, x-velocity and y-velocity (from left to right) on the free surface around a cylinder is shown in the following figures. The first quadrant of an annulus $1 < R < 2$ is shown.

The errors are the normalized (by maximum pressure or velocity on the free surface patch) difference between those obtained as WAMIT output at specified field points and those obtained from the approximation with B-splines using the value at **NF** collocation points (in B-splines approximation of the pressure/velocity).

Other notations are described below:

The incident wave heading is 0 and the body is freely floating and free to move. The cylinder's dimensions are $R=1$ and $D=2$.

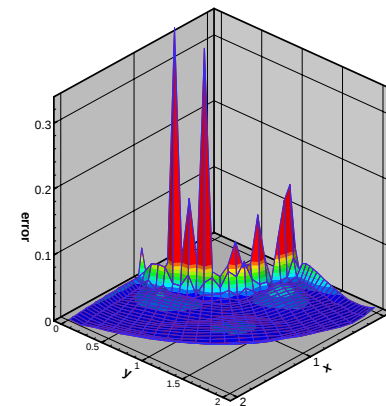
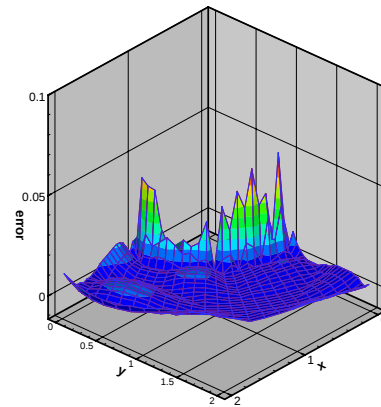
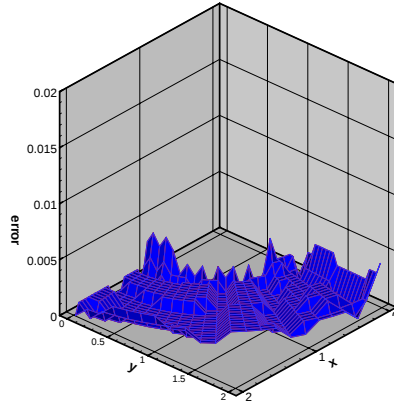
KR: wave number

NU: number of panels in azimuthal direction (for approximation on free surface)

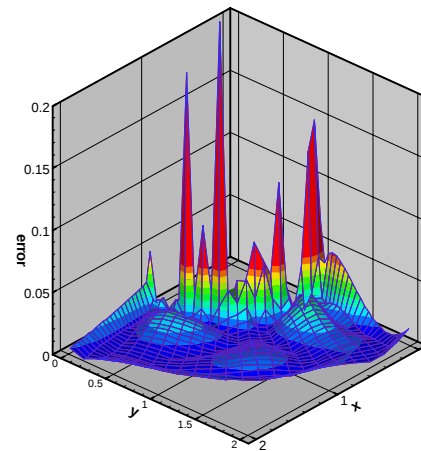
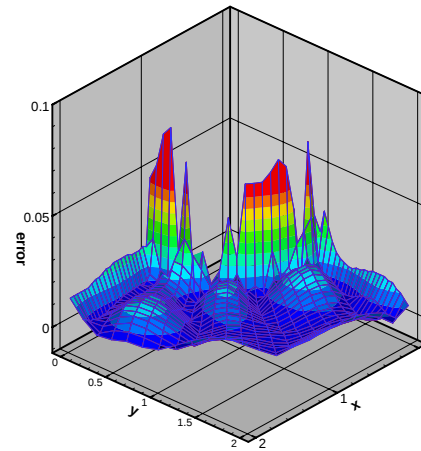
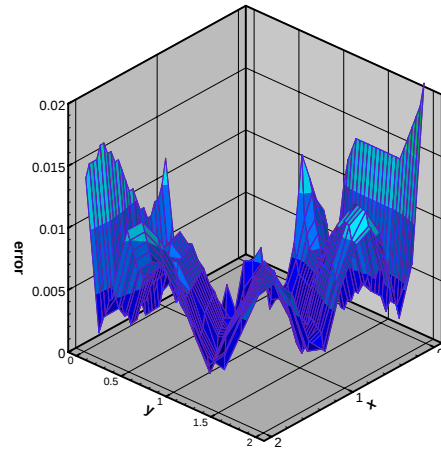
NV :number of panels in radial direction

NF: number of collocation points

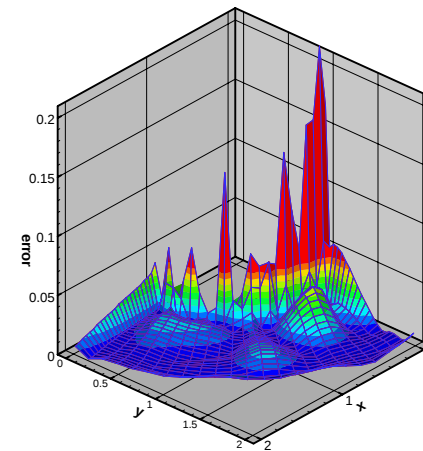
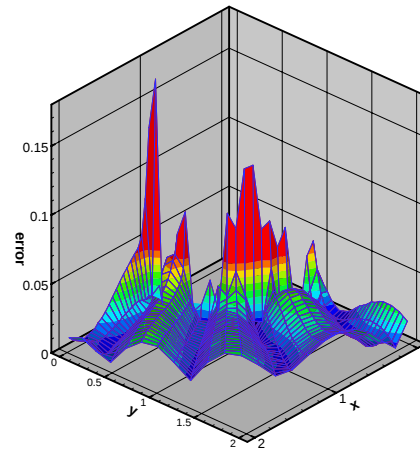
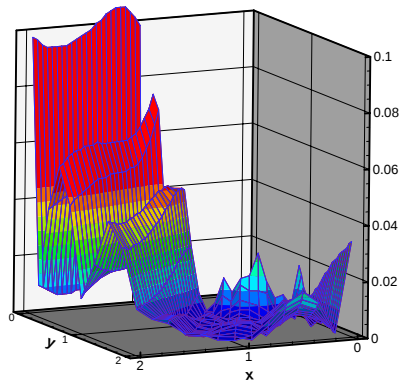
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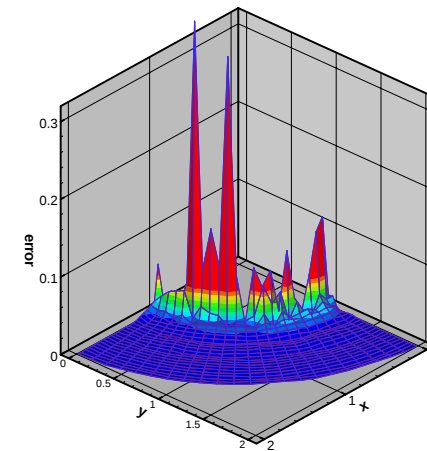
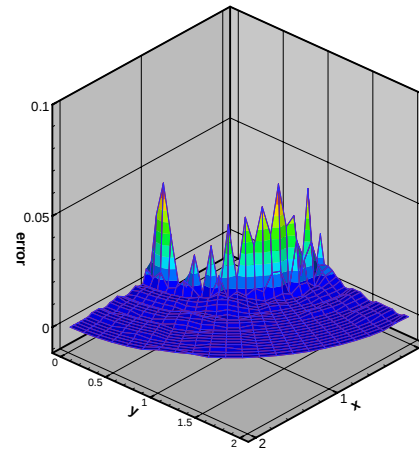
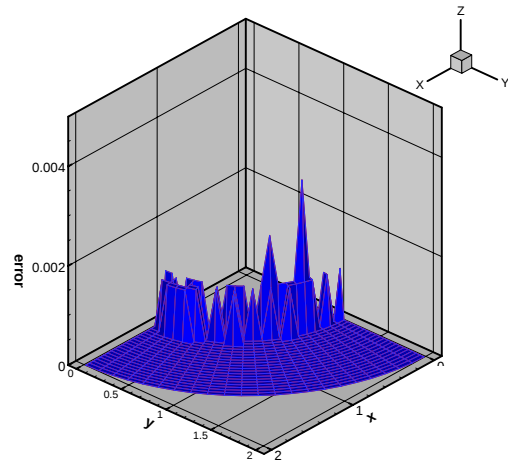
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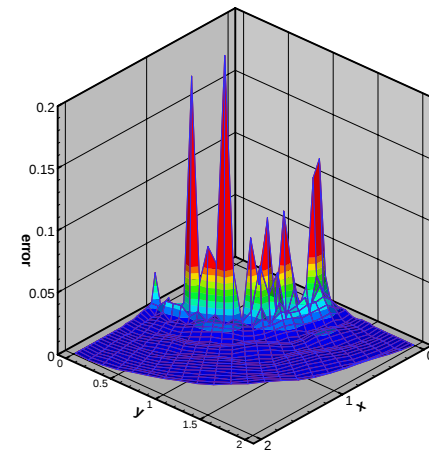
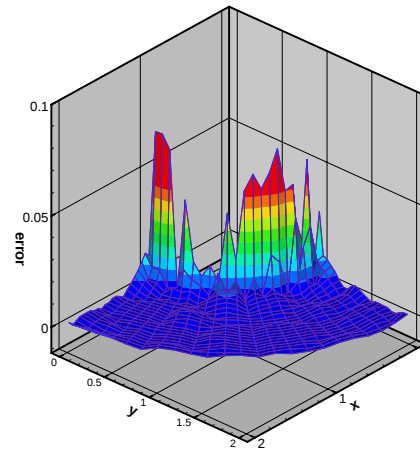
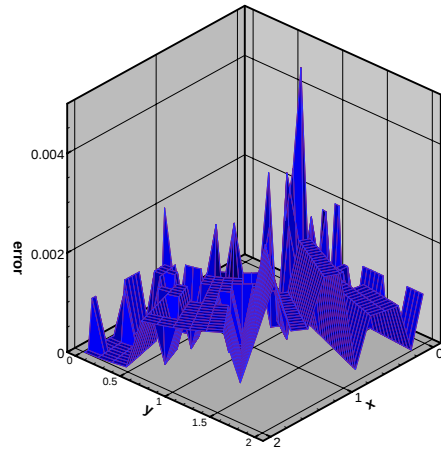
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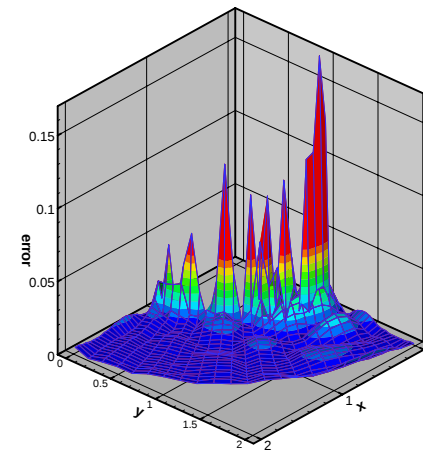
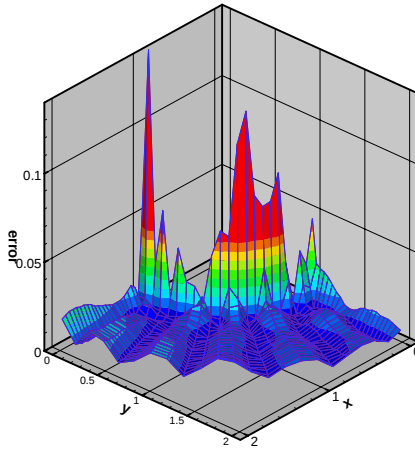
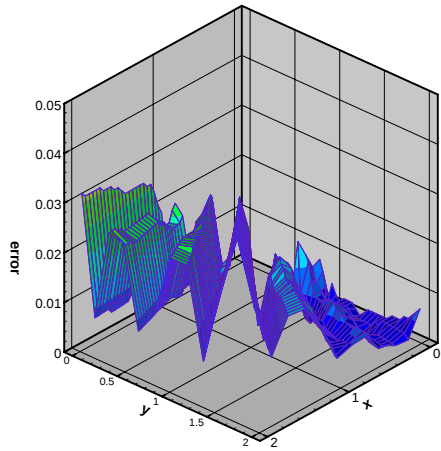
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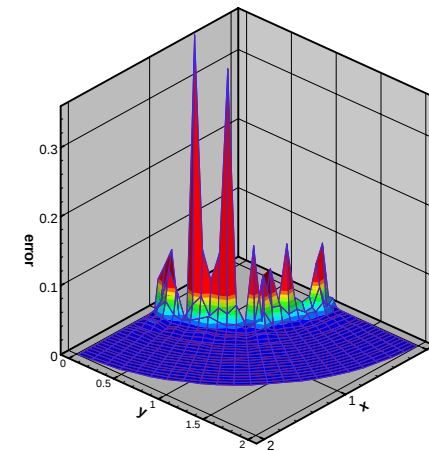
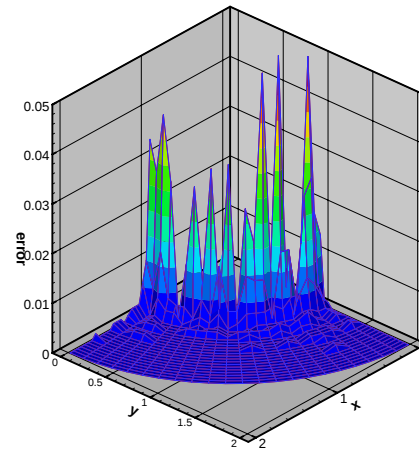
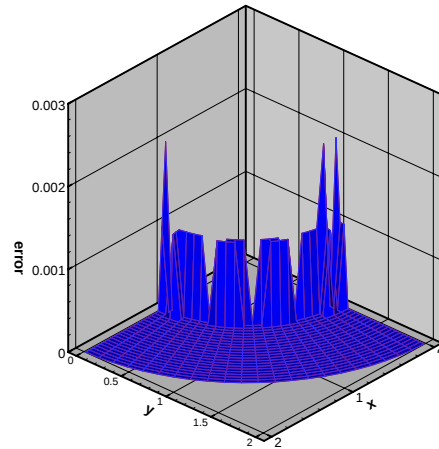
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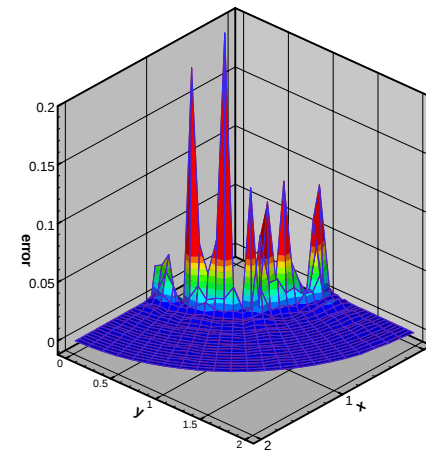
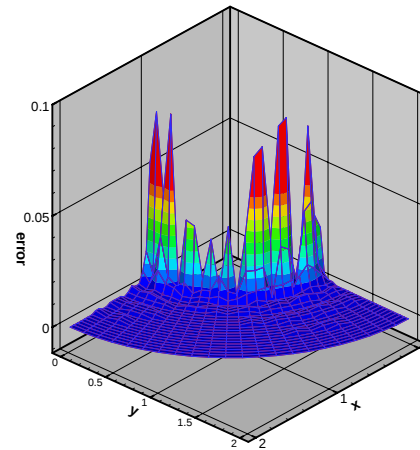
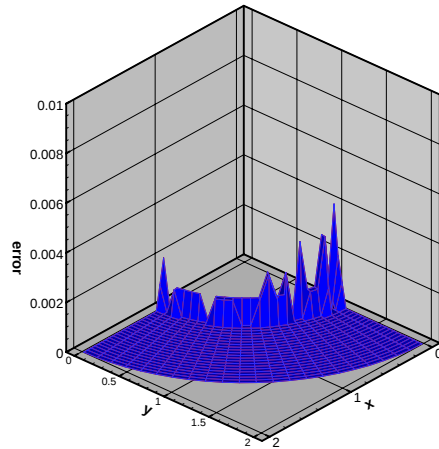
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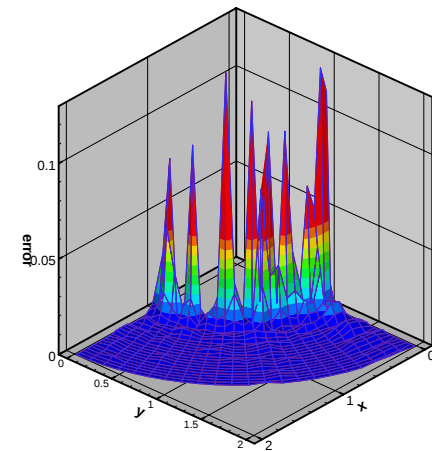
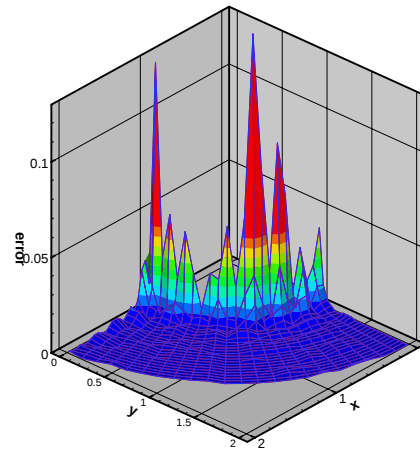
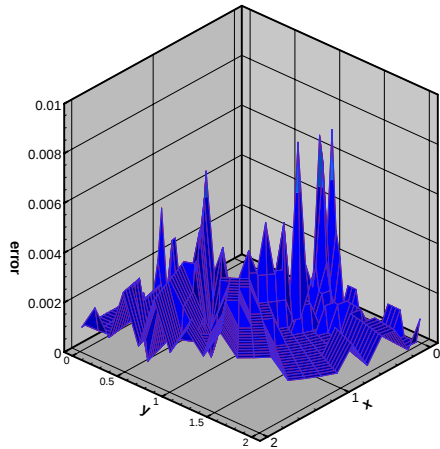
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KR=1., NU=8, NV=4, NF=288



KR=2, NU=8, NV=4, NF=288



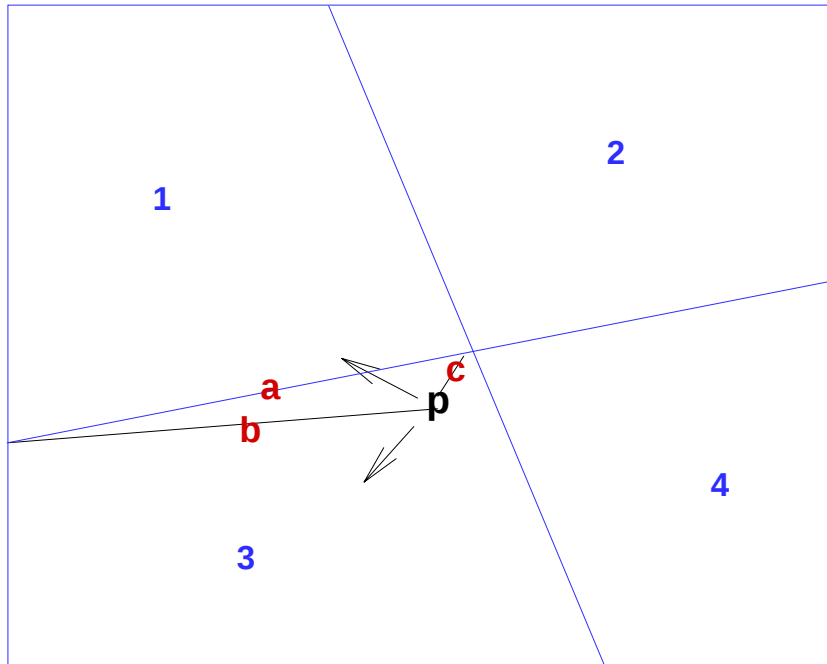
Summary

- The pressure and velocity are evaluated accurately by the approximation at most field points.
The approximation improves consistently with the increased use of collocation points.
- Large errors are found in the velocity in the close vicinity of the waterline ($1 < R < 1.01$). These local errors appear not sensitive to the change in the collocation points.
- Since the quantities of interest are the integrated values of the forcing over the free surface, the local errors in the velocity may not have significant effects. This needs to be confirmed and works are in progress.

Automatic shifting of Free Surface Field points

- When field points are close to the edge of the free surface panels, the program may stop due to the error in the evaluation of Rankine source or the computed results may be inaccurate.
- In order to avoid the interruption and to have more accurate results, users have to choose input field points carefully, typically near the centroids of the free surface panels.
- A method is developed to identify the closeness of the field points to the edge and move the point away from the latter. This extension would allow users to input the field points without concerning the discretization of the free surface.

Method



P: field point

1-4: panel

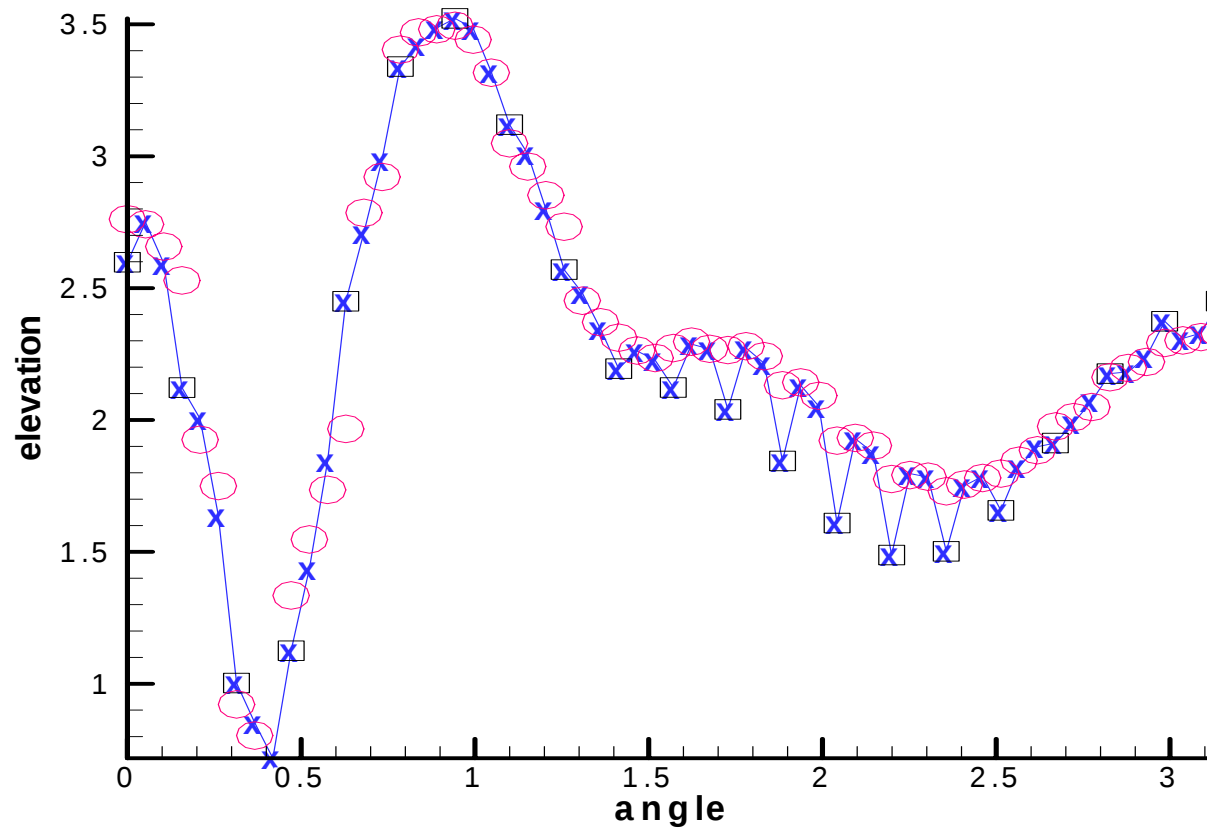
a, b, c: length

- 1 For P find 4 panels small distance from one of its vertices (panels 1, 2, 3, 4)
- 2 Compare length of panel sides a with $b + c$ (panels 1 & 3) (If $b+c-a > \text{tolerance}$ no shift needed)
- 3 Move P toward centroids of 1 & 3 and choose the direction $b+c$ increases

Double-frequency free surface elevation around a bottom mounted cylinder of radius 1 and draft 1 from lee to weather sides. (The 2nd-order wave number $KR=3.2$.)

x with boxes: program interrupted. Then points moved toward centroids, 0.01 of distance
x: without boxes: points not shifted

○: all points moved toward centroids, 0.5 of the distance



Summary

A method is implemented to identify the points near the edge and to shift away from the edge in order to avoid the interruption of the run or inaccurate evaluation of the free surface elevation/field-pressure.

The parameter used for the points near the edges is $1E-2$ from the edge when normalized by the maximum value of 4 sides.

These points are moved half way toward the centroids of the corresponding panels.

The indices of the effected points are output in ERRORF.LOG.
New coordinates of the field points, after shifting, are output in .FPT file. (The original coordinates are in .FRC)

Progress with trimmed waterlines
(using the higher-order method)

By J. N. Newman

Background

Work was described at the last meeting on trimming of waterlines, using both the low-order and higher-order methods. For the low-order method this is straightforward, and essentially completed.

For the higher-order method only limited cases were considered, where the waterline intersects only once on two opposite or adjacent sides and where the submerged portion of each patch can be mapped onto a single corresponding rectangular patch in parametric coordinates using a regular mapping. This work has been generalized, to support a wider range of cases.

General Approach

(essentially the same as last year **except where noted in red**)

- Body geometry must be defined up to at least the plane of the free surface, and may extend above this plane
- Geometry, and hydrodynamic outputs, are defined relative to conventional body-fixed coordinates
- New parameter ITRIMWL=0 (default) or **>0** in config file
- New array XTRIM in config file, specifying the vertical displacement (heave) and trim (pitch, roll in degrees) relative to the origin of the body coordinates
- Pitch and roll are Euler angles, in that order
- WAMIT trims the waterline, including only the portion of the geometry below $Z=0$ (global plane of free surface)
- **When necessary patches are subdivided into two or more new patches with regular mapping on each**

Higher-order Approach (ILOWHI=1)

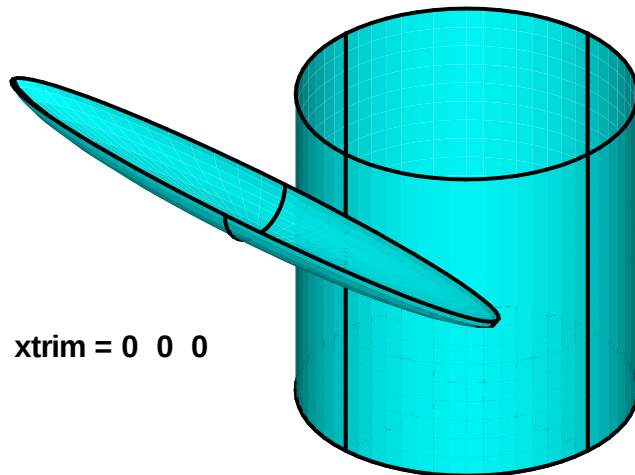
- First check all patches, eliminate if `dry' and tag waterline patches which span $Z=0$
- For waterline patches the computational domain (parametric) $(U,V)=(-1,1)$ is mapped to the submerged portion of the patch
- If submerged portion is `rectangular' a ruled mapping is used
- If submerged portion is `triangular', a singular point is introduced at the submerged vertex
- In other cases, subdivide into two or more patches.
- NPATCH is reduced for dry patches or increased for subdivided patches

Special Points

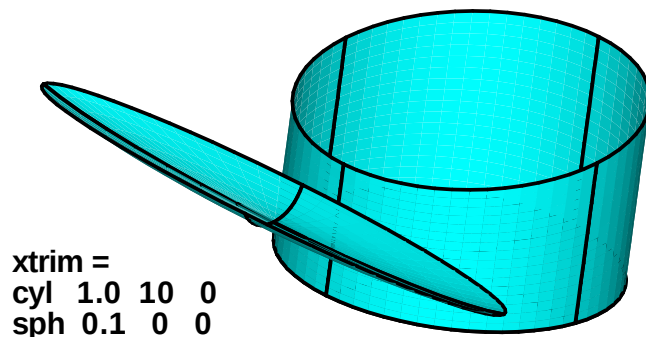
- If $ITRIMWL > 0$ the error message regarding panel/patch vertices above the free surface is disabled
- $IRR=1$ requires user to represent interior free surface (awkward)
- $IRR=2$ (projection of panels onto free surface) may be affected by pitch and roll displacements. (Should be OK with $IRR=2,3$ options, but not yet tested.)
- Angular displacements may affect symmetry. WAMIT automatically reflects when this is necessary, as in the examples shown below.
- Internal tank waterlines are not trimmed. Special attention is required for tank free surfaces if angular displacements are included.
- Trimming of higher-order patches could fail if the trace of the waterline is irregular in parametric space

Examples shown last year (ILOWHI=1)

NB: All perspectives show only the submerged portion of the body

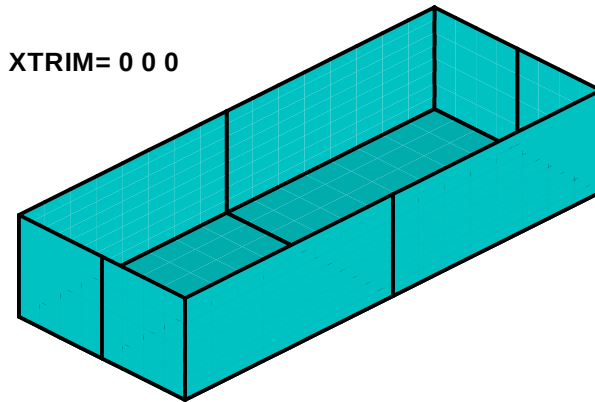


xtrim = 0 0 0

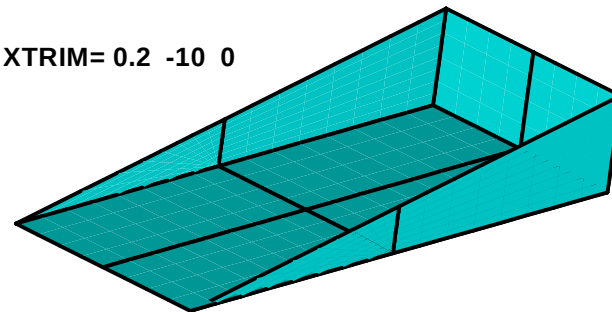


xtrim =
cyl 1.0 10 0
sph 0.1 0 0

XTRIM= 0 0 0



XTRIM= 0.2 -10 0

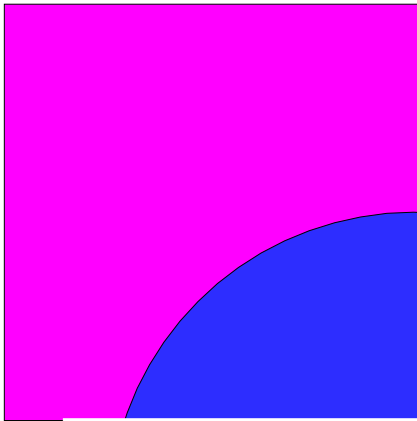


Patch topologies in parametric space (U,V)

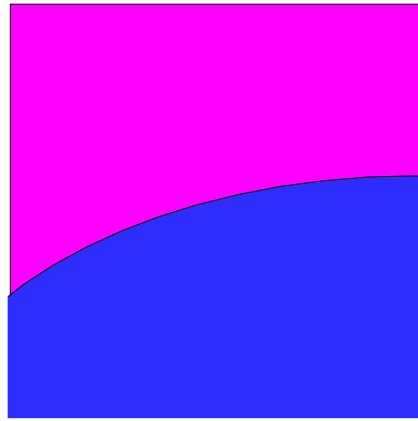
Blue = wet surface **Red = dry surface**

Cases included last year

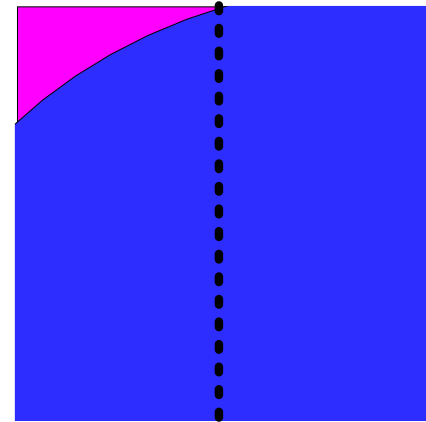
Type 1



Type 2

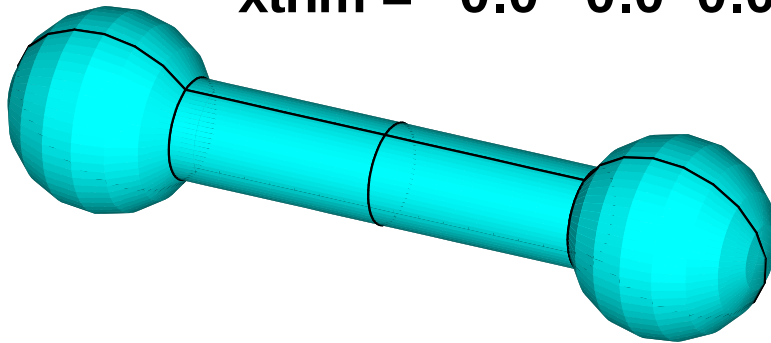


Type 3

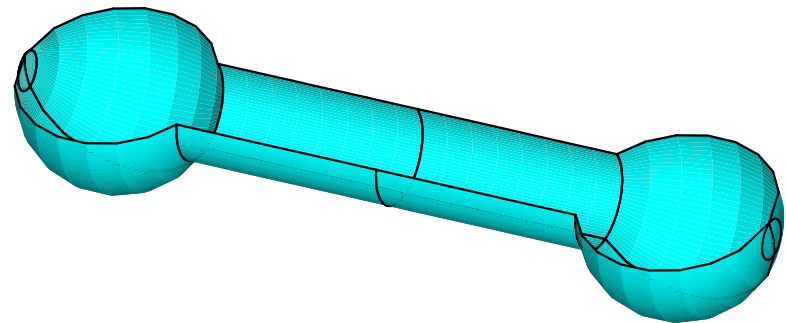


`Dumbbell', with vertical trim
RADCYL=0.5, RADSPH=1.0, DAXIS=2.0
(varying vertical trim, no pitch or roll)

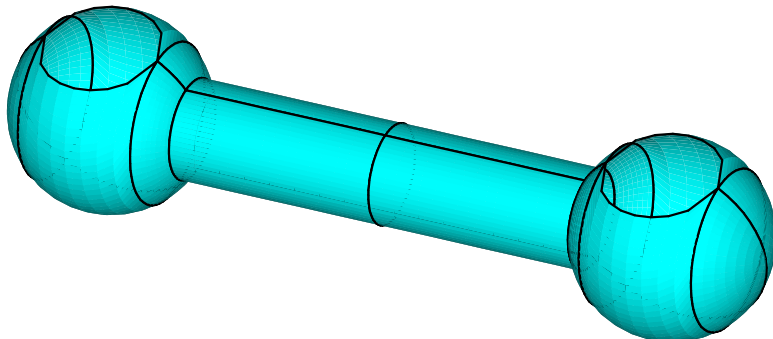
xtrim = 0.0 0.0 0.0



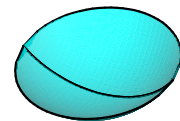
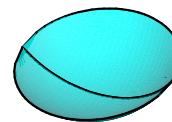
xtrim = 1.8 0.0 0.0



xtrim = 1.2 0.0 0.0

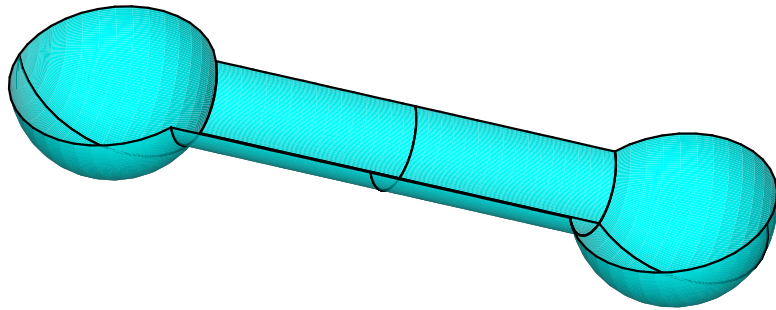


xtrim = 2.6 0.0 0.0

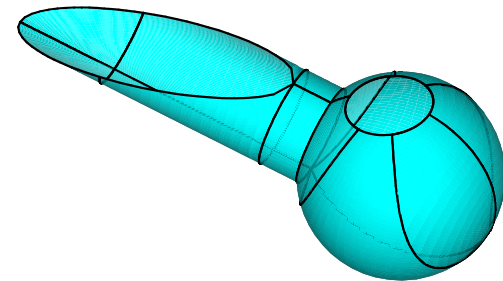


`Dumbbell', with vertical trim and pitch angle
RADCYL=0.5, RADSPH=1.0, DAXIS=2.0
(pitch = 0, 10, 20, 30 deg)

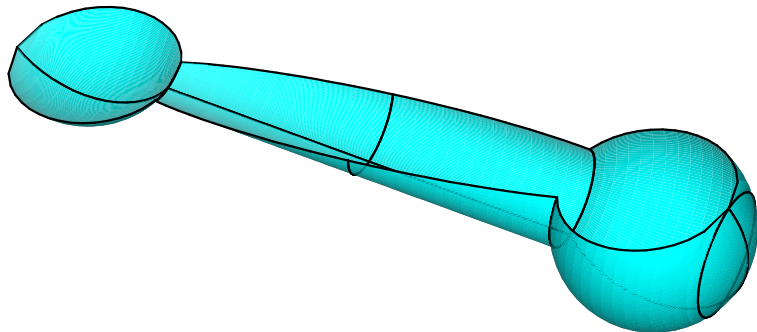
xtrim = 2.0 0.0 0.0



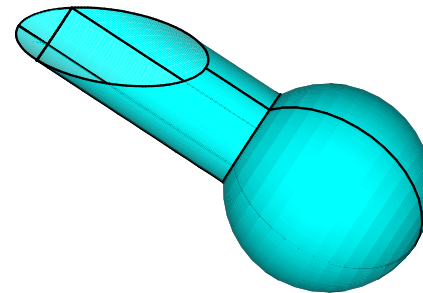
xtrim = 2.0 20.0 0.0



xtrim = 2.0 10.0 0.0



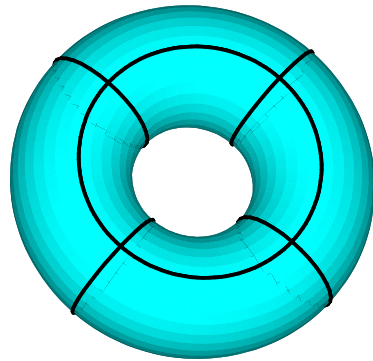
xtrim = 2.0 30.0 0.0



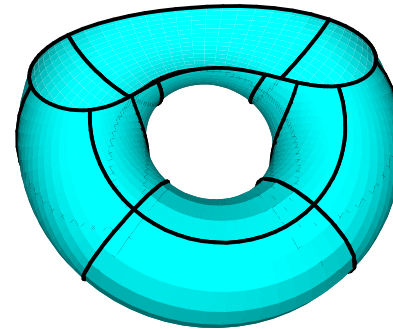
Torus (IGDEF=-8)

First figure is completely submerged, others are raised by xtrim(1)

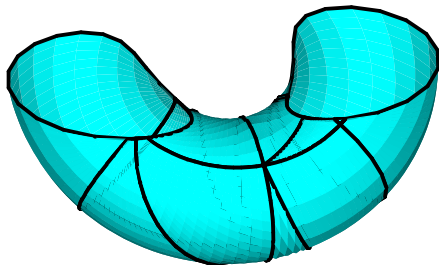
xtrim = 0.0 30.0 30.0



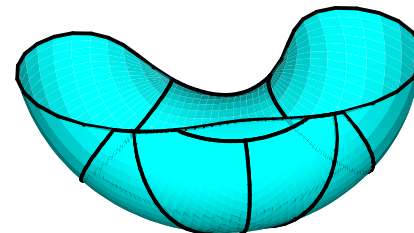
xtrim = 1.0 30.0 30.0



xtrim = 1.6 30.0 30.0

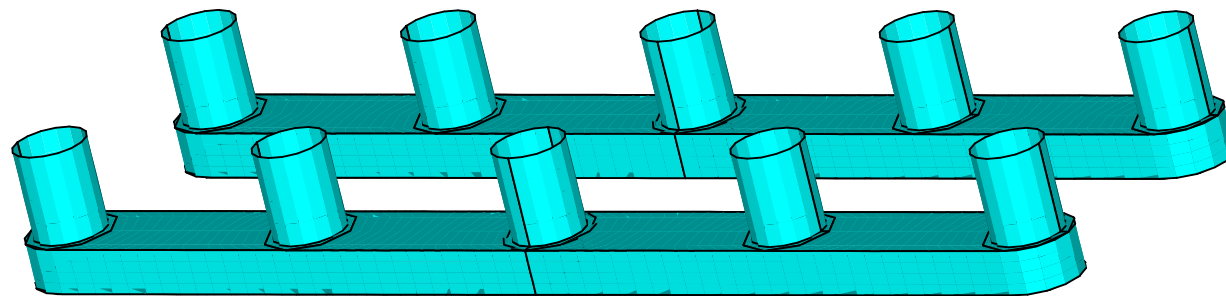


xtrim = 1.7 30.0 30.0

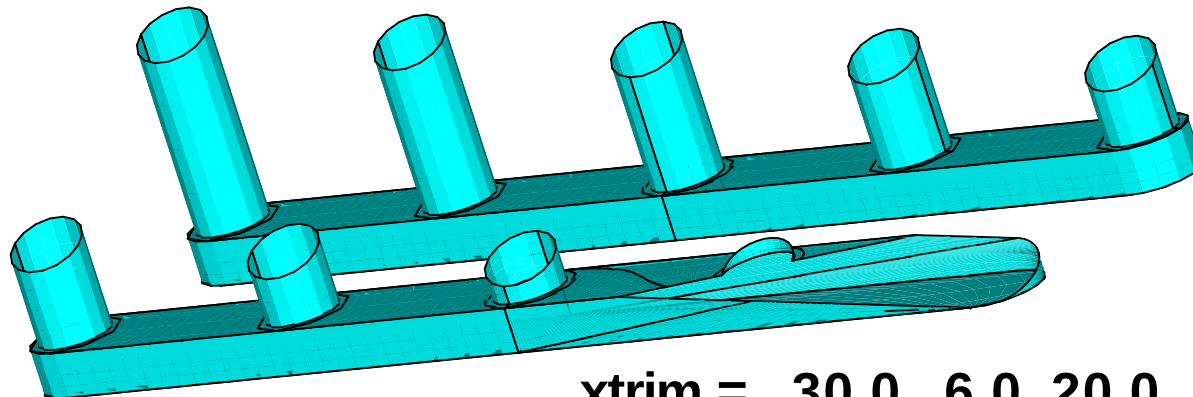


Semisub (IGDEF=-10)

(Top figure is untrimmed, bottom figure at 6 deg pitch, 20 deg roll)



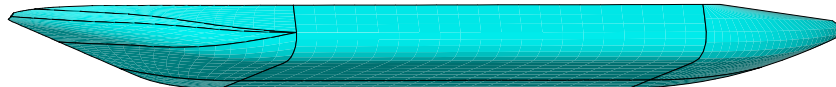
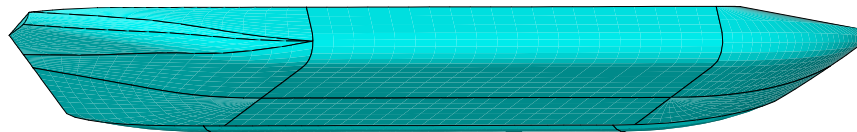
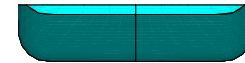
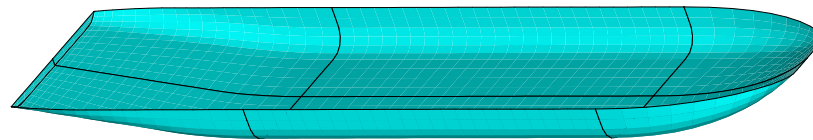
xtrim = 30.0 0.0 0.0



xtrim = 30.0 6.0 20.0

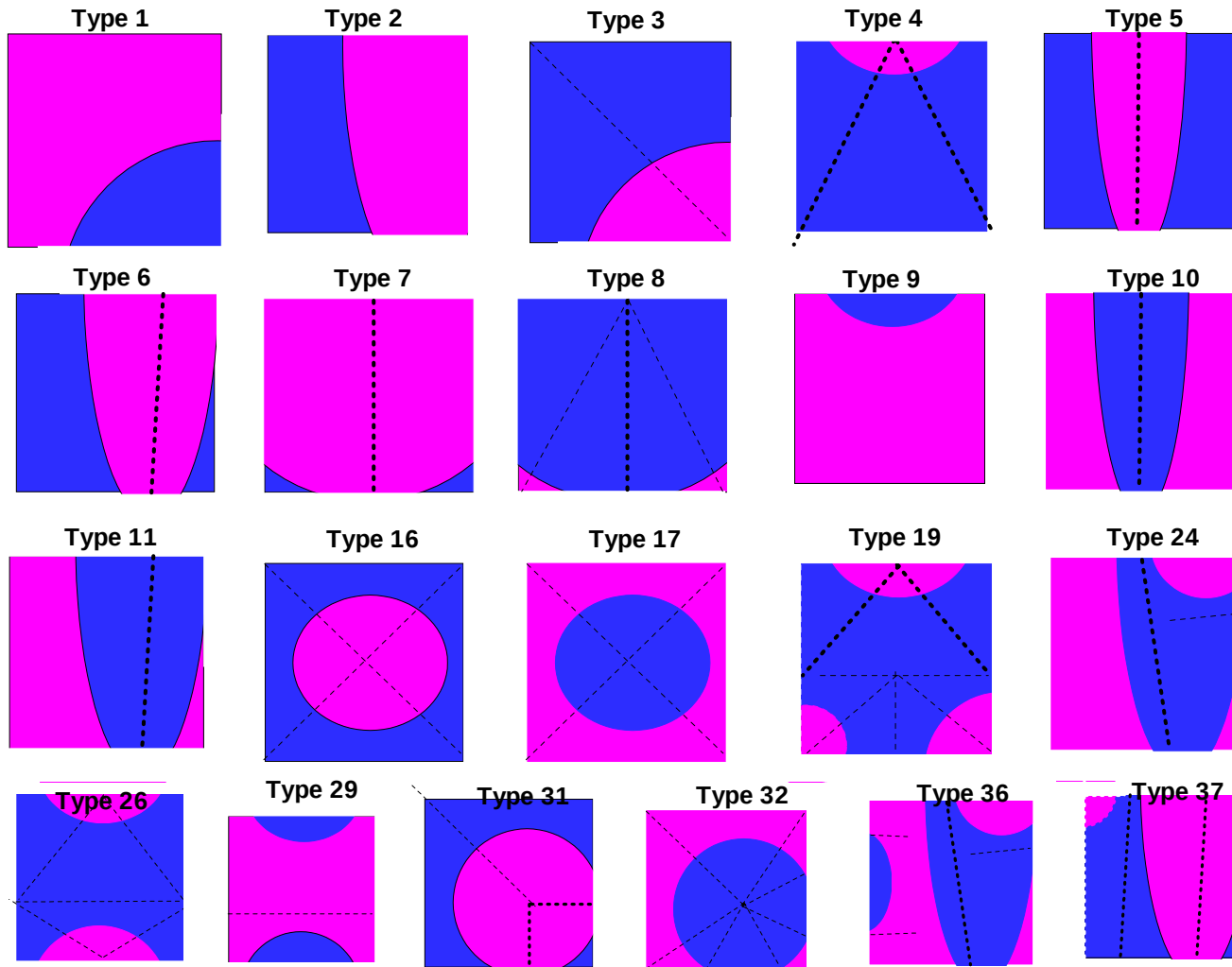
MultiSurf FPSO (Test 20)

Roll = 0,10,20,30 degrees about port side



Topologies in parametric space (U,V) for the examples above

Blue = wet surface Red = dry surface



Present Status (ILOWHI=1)

- Geometry trimming has been coded (37 types)
- These are `ad hoc'
- Current work is on iterative subdivision of patch to achieve greater generality
- Patch subdivision is complicated from the coding standpoint (must use both original GDF patch indices and new indices)
- Not yet implemented in WAMIT beyond input and trimming of geometry and output of .dat files

Evaluation of gravitational moments for generalized modes

Definition of the hydrostatic coefficients in WAMIT (Output in HST file.)

- Six degrees of freedom when specified as MODES 1, 2,.. 6 in POT:
The hydrostatic restoring force coefficients corresponding to these modes include the changes in the buoyancy and gravitational forces
- Generalized modes specified using NEWMODES or DEFINE.F:
The hydrostatic coefficients computed for these modes include only buoyancy force.

The restoring forces due to gravity should be specified as the external restoring force matrix as an input in addition to other restoring forces.

Left column: the hydrostatic coefficients output for 6 modes

Right column: the hydrostatic coefficients output for the same 6 modes when described as generalized modes

Complete forces

$$c(3, 3) = \rho g \iint_{S_b} n_3 dS$$

$$c(3, 4) = \rho g \iint_{S_b} y n_3 dS$$

$$c(3, 5) = -\rho g \iint_{S_b} x n_3 dS$$

$$c(4, 2) = 0$$

$$c(4, 4) = \rho g \iint_{S_b} y^2 n_3 dS + \rho g \forall z_b - m g z_g$$

$$c(4, 5) = -\rho g \iint_{S_b} x y n_3 dS$$

$$c(4, 6) = -\rho g \forall x_b + m g x_g$$

$$c(5, 1) = 0$$

$$c(5, 5) = \rho g \iint_{S_b} x^2 n_3 dS + \rho g \forall z_b - m g z_g$$

$$c(5, 6) = -\rho g \forall y_b + m g y_g$$

$$c(i, j) = c(j, i)$$

except

$$c(6, 4), c(6, 5) = 0$$

Buoyance forces

$$c(3, 3) = c(3, 3)$$

$$c(3, 4) = c(3, 4)$$

$$c(3, 5) = c(3, 5)$$

$$c(4, 2) = -\rho g \forall$$

$$c(4, 4) = \rho g \iint_{S_b} y^2 n_3 dS + \rho g \forall z_b$$

$$c(4, 5) = c(4, 5)$$

$$c(4, 6) = -\rho g \forall x_b$$

$$c(5, 1) = \rho g \forall$$

$$c(5, 5) = \rho g \iint_{S_b} x^2 n_3 dS + \rho g \forall z_b$$

$$c(5, 6) = -\rho g \forall y_b$$

$$c(i, j) = c(j, i)$$

except

$$c(2, 4), c(1, 5), c(6, 4), c(6, 5) = 0$$

In the following equations of motion, **M**, **B** and **C** denote inertia, damping force and stiffness matrices, respectively. These are input to FORCE module.

The gravity forces are to be added to **C**.

$$\sum_{j=1} \xi_j [-\omega^2 (\mathbf{M}_{ij} + a_{ij}) + i\omega (b_{ij} + \mathbf{B}_{ij}) + c_{ij} + \mathbf{C}_{ij}] = X_i$$

Example 1 : when 6 rigid body modes are described as the generalized modes using NEWMODES (as modes 7 to 12). C matrix should have the following nonzero elements

$$C(10, 8) = mg$$

$$C(10, 10) = -mgz_g$$

$$C(10, 12) = mgx_g$$

$$C(11, 7) = -mg$$

$$C(11, 11) = -mgz_g$$

$$C(11, 12) = mgy_g$$

$$\text{All other elements, } C(i, j) = 0$$

Example 2 : When both bending and torsional responses of the structure are considered at the same time, **C** matrix should include the following forces (corresponding to C(4,2) and C(4,4) in 2D) in addition to the structural restoring force.

$V_i(x)$: vertical mode shapes

$H_i(x)$: horizontal mode shapes

$\theta_i(x)$: torsional mode shapes

$$C_{ij} = g \int \rho(x) H_j(x) \theta_i(x) dx$$

$$C_{ij} = -g \int \rho(x) \theta_i(x) \theta_j(x) z_g(x) dx$$

Nontrivial coupling of surge and pitch with vertical modes should be considered.

Also the coupling of sway, roll and yaw with the torsional modes should be considered.

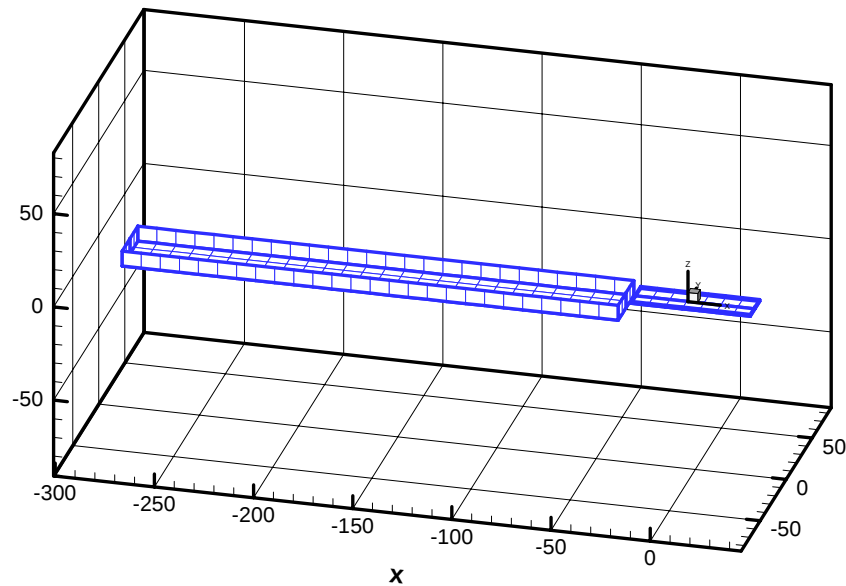
When heaviside step function modes are used to calculate the shear force or bending moment using the fixed mode option, the effect of the gravitational moments to these modes due to free modes should be included.

Example 3: Two barges connected with a hinge

barge 1: $L=60\text{m}$ $B=18\text{m}$ $D=1\text{m}$

barge 2: $L=250\text{m}$ $B=30\text{m}$ $D=8\text{m}$

Gap : 5m between two barges



Hinge mode

$$\frac{\partial \phi_7}{\partial n} = \begin{matrix} (z - z_h)n_1 - (x - x_h)n_3 & \text{on barge2} \\ 0 & \text{on barge1} \end{matrix}$$

Fixed modes

j	description	$\frac{\partial \phi_j}{\partial n}$ on barge 2	vertical displacement
8	vertical shear force	n_3	1
9	horizontal shear force	n_2	0
10	tension/compression	n_1	0
11	tortional moment	$yn_3 - (z - z_h)n_2$	y
12	horizontal bending moment	$(x - x_h)n_2 - yn_1$	0

Mass matrix

$$M_{ij} = M_{ji} = \int \rho(\zeta_i \cdot \zeta_j) dv$$

$$\zeta_1 = \hat{i}$$

$$\zeta_2 = \hat{j}$$

$$\zeta_3 = \hat{k}$$

$$\zeta_4 = y\hat{k} - z\hat{j}$$

$$\zeta_5 = z\hat{i} - x\hat{k}$$

$$\zeta_6 = x\hat{j} - y\hat{i}$$

$$\zeta_7 = (z - z_h)\hat{i} - (x - x_h)\hat{k} \text{ for } x < x_h \text{ and } \zeta_7 = 0 \text{ for } x > x_h$$

$$\zeta_8 = \hat{k} \text{ for } x < x_h \text{ and } \zeta_8 = 0 \text{ for } x > x_h$$

$$\zeta_9 = \hat{j} \text{ for } x < x_h \text{ and } \zeta_9 = 0 \text{ for } x > x_h$$

$$\zeta_{10} = \hat{i} \text{ for } x < x_h \text{ and } \zeta_{10} = 0 \text{ for } x > x_h$$

$$\zeta_{11} = (y - y_h)\hat{k} - (z - z_h)\hat{j} \text{ for } x < x_h \text{ and } \zeta_{11} = 0 \text{ for } x > x_h$$

$$\zeta_{12} = (x - x_h)\hat{j} - (y - y_h)\hat{i} \text{ for } x < x_h \text{ and } \zeta_{12} = 0 \text{ for } x > x_h$$

(Subscript 'h' denotes the coordinates of the hinge)

Nonzero elements in the stiffness matrix

$$C_{7,1} = -m_2 g$$

$$C_{11,2} = m_2 g$$

$$C_{11,4} = -m_2 g z_{g2}$$

$$C_{7,5} = -m_2 g z_{g2}$$

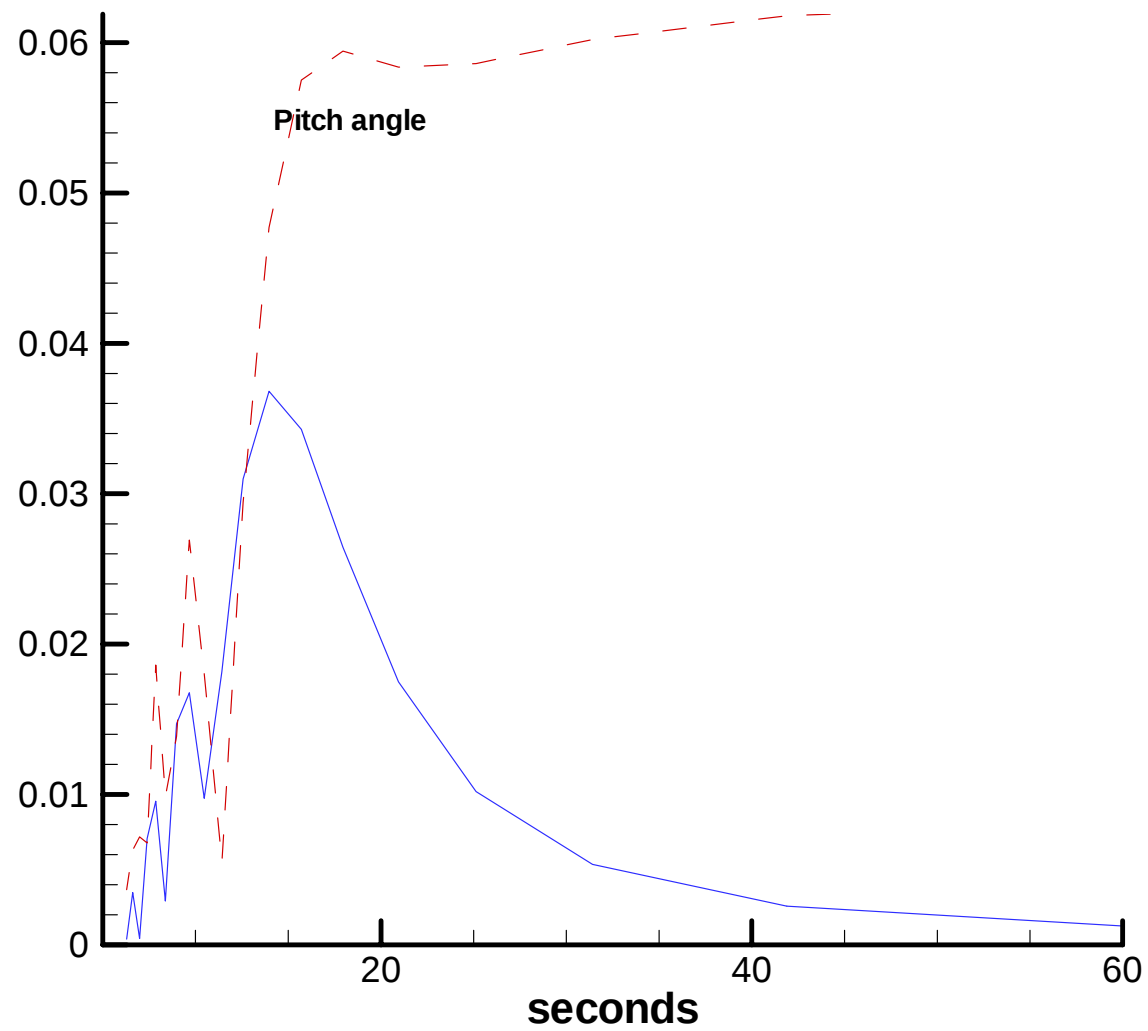
$$C_{5,7} = C_{7,7} = -m_2 g (z_{g2} - z_h)$$

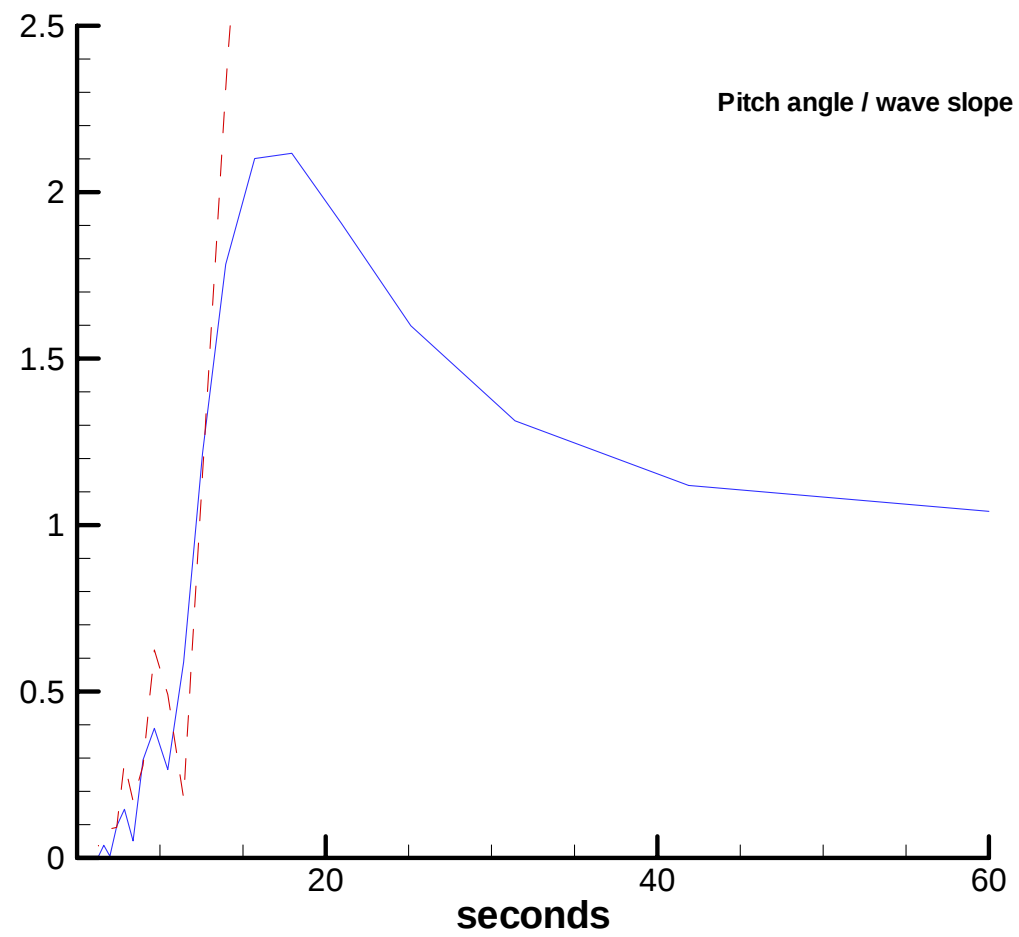
$$C_{11,6} = m_2 g x_{g2}$$

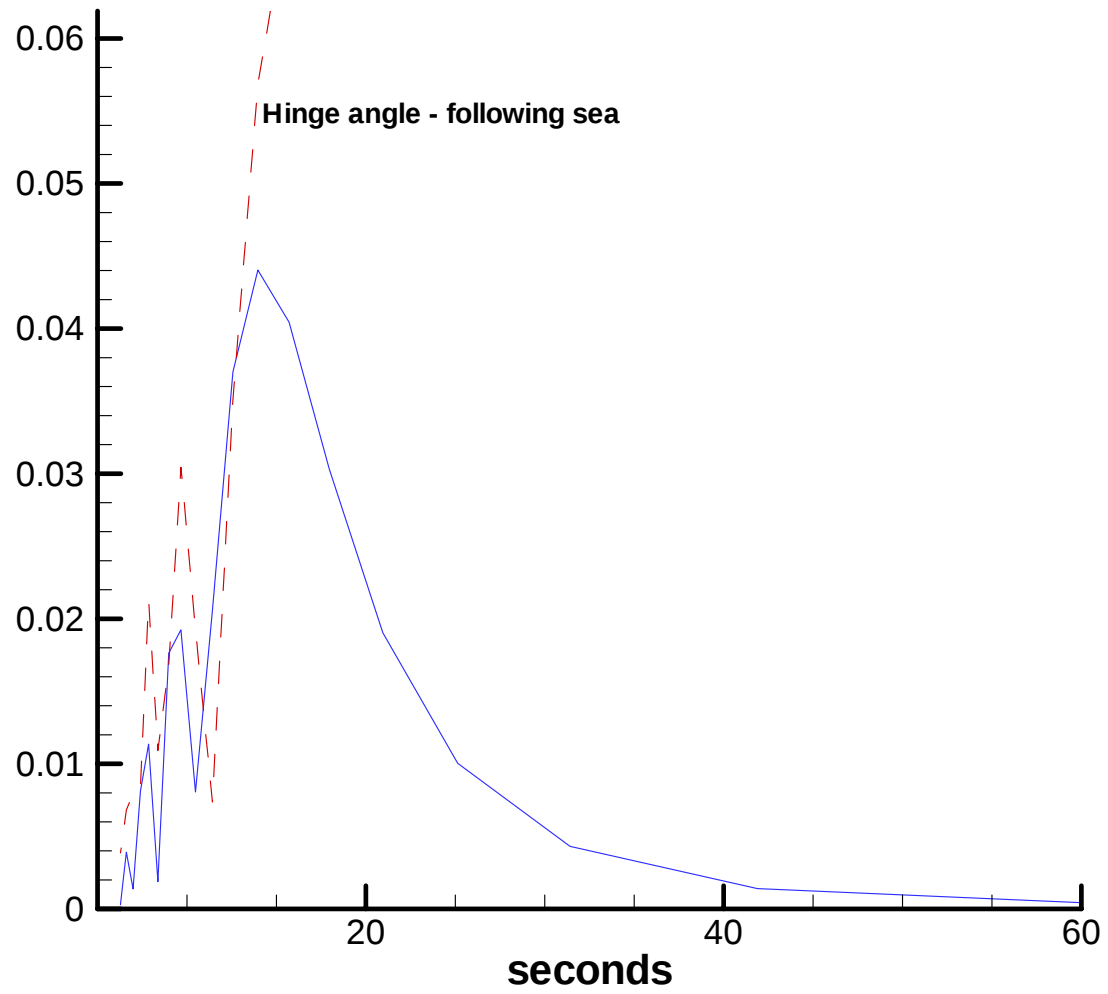
$$C_{7,6} = m_2 g y_{g2} = 0$$

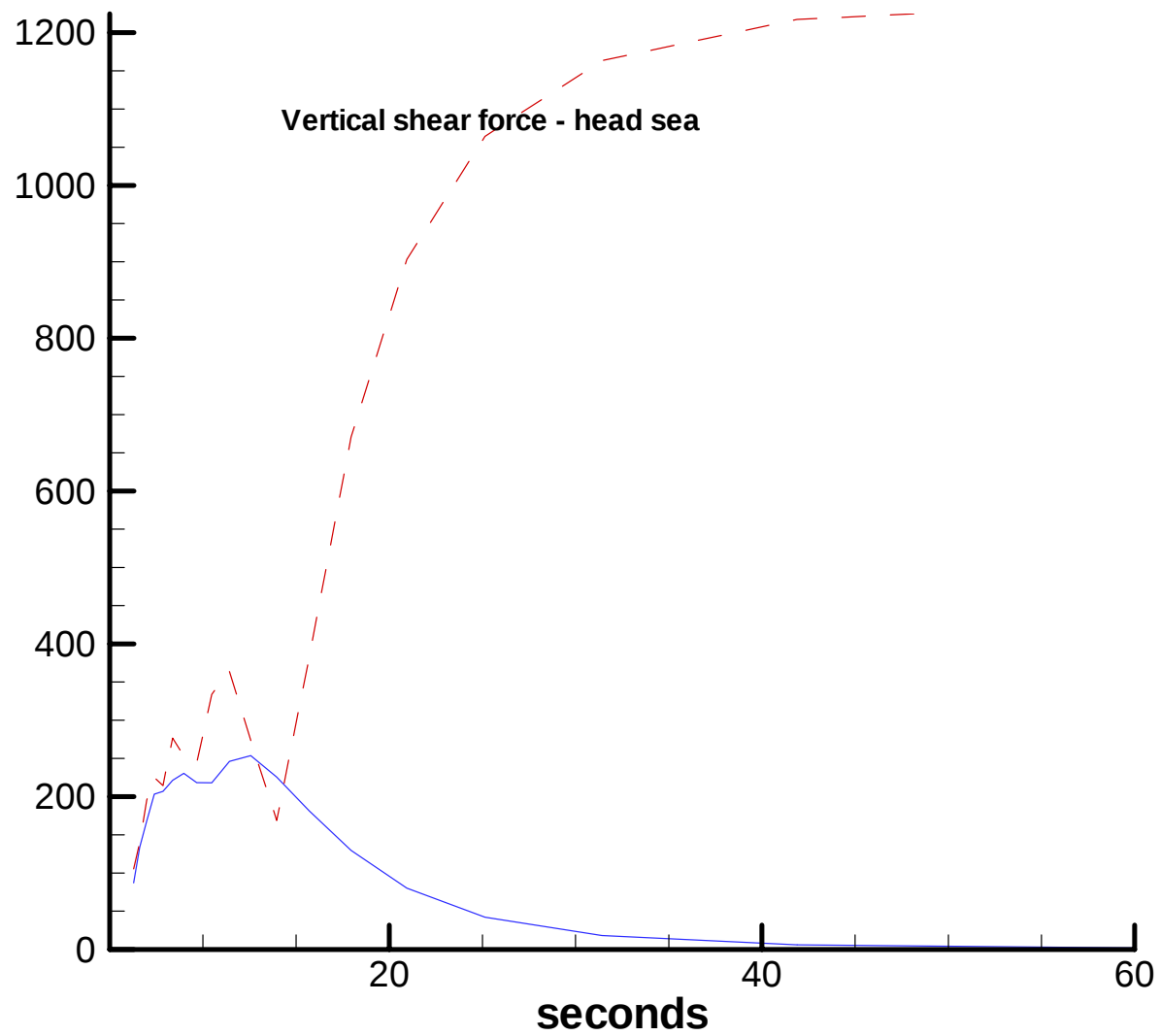
(Subscript '2' denotes barge 2.)

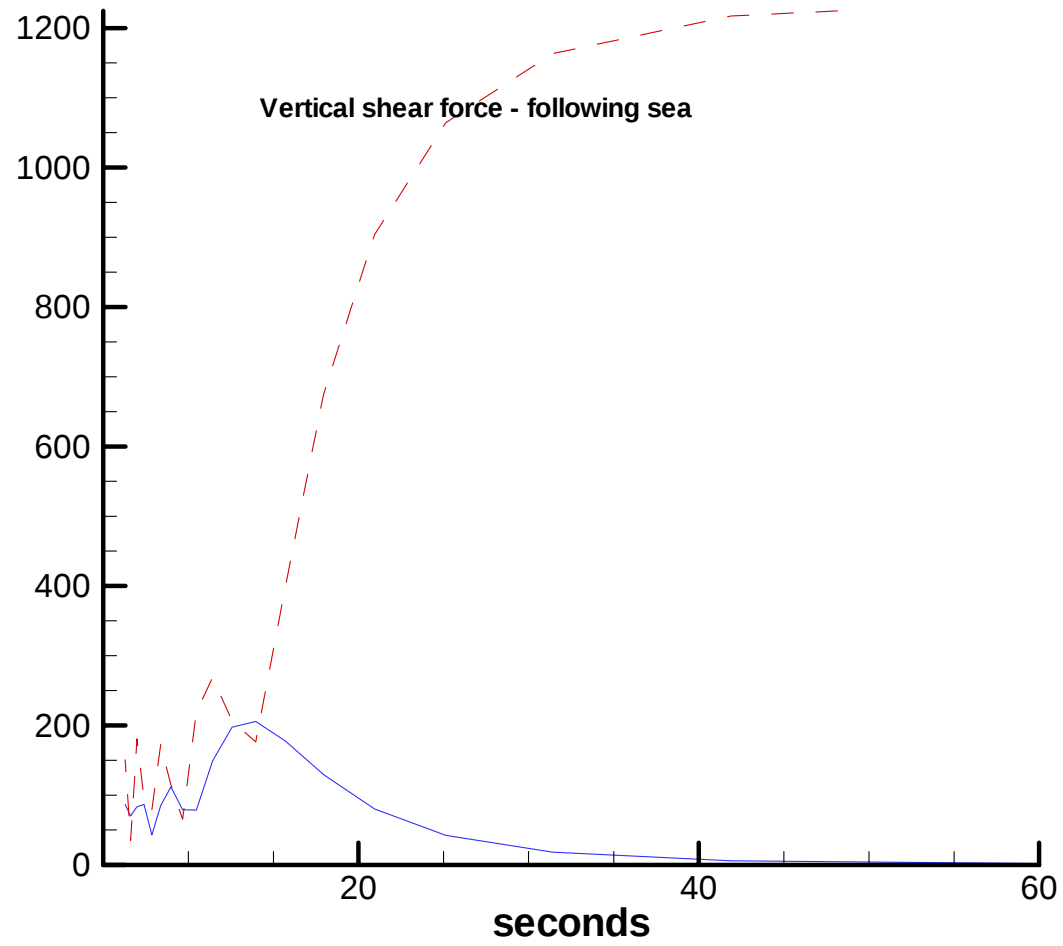
Solid line : with restoring force
Dashed line : without restoring force

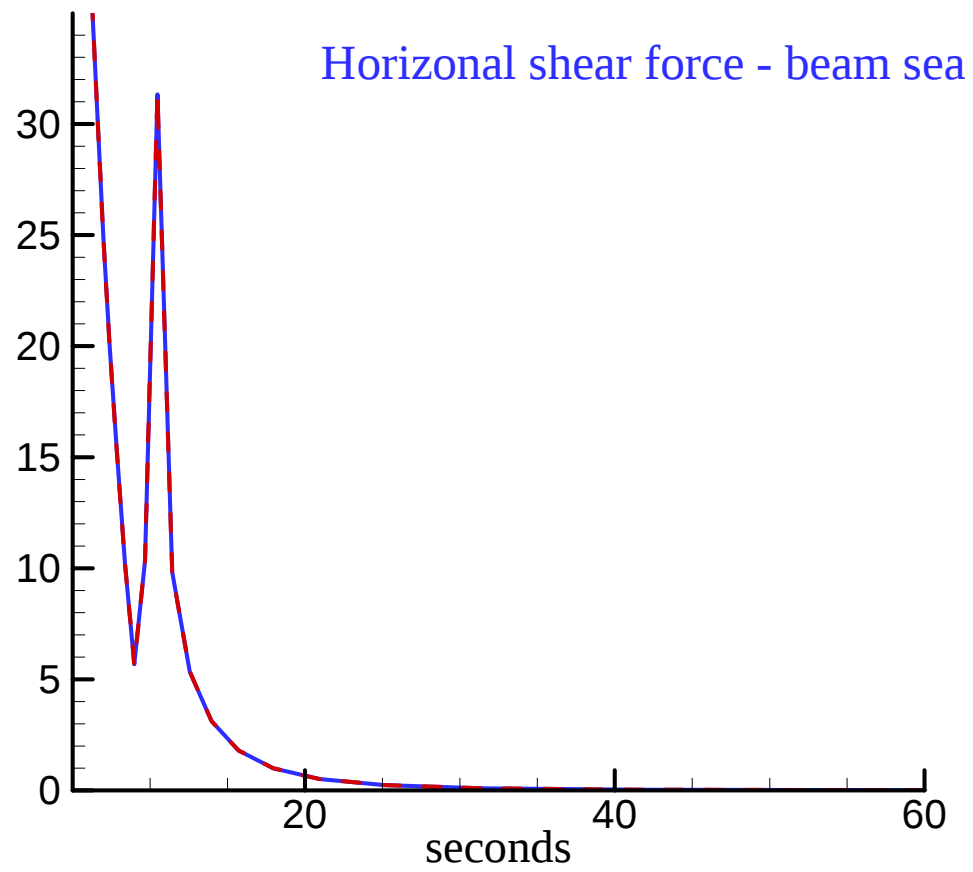


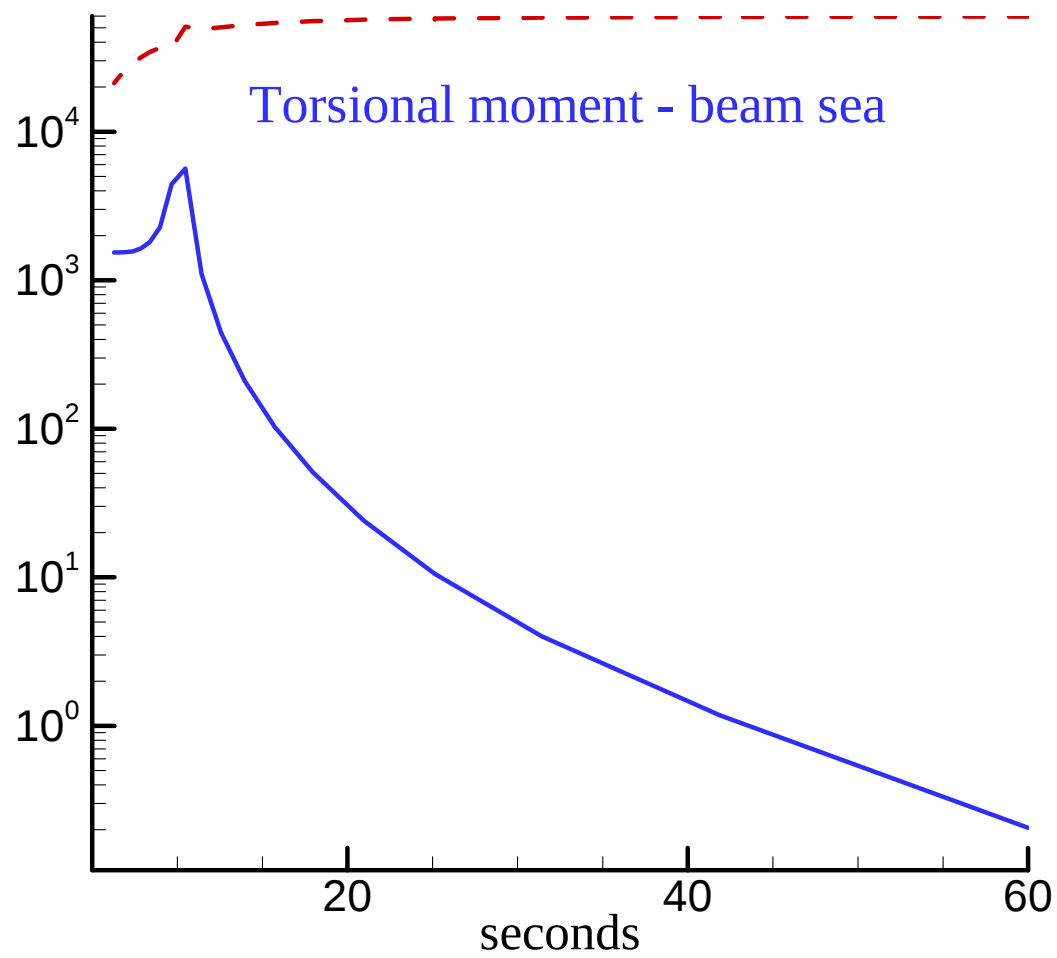












Summary

- a) The restoring forces due to gravity should be included in the external restoring force (or stiffness) matrix as an input (with other restoring forces).
- b) When neglected, the error in the computational results is apparent in the long waves when inertial force becomes relatively small.
- c) The gravitational restoring force affects the moments only and, thus the stiffness matrix is not symmetric.
- d) Here we consider the rotation of one barge for the hinge mode. When other mode shapes are used, the restoring forces should be evaluated in an appropriate manner. For example, when the hat function is used for the hinge mode, the coupled buoyancy force with surge (computed in WAMIT) and the gravitational moment to be included in the stiffness matrix are, respectively, as follows.

$$c_{7,1} = \rho g \int n_1 w_7 dv = \rho g \left[\int_{b_1} n_1 \left(\frac{-x}{L_1} + 1 \right) dv + \int_{b_2} n_1 \left(\frac{x}{L_2} + 1 \right) dv \right] = \rho g \left(\frac{\forall_2}{L_2} - \frac{\forall_1}{L_1} \right)$$

$$C_{7,1} = \frac{m_1 g}{L_1} - \frac{m_2 g}{L_2}$$

(w_7 is the vertical displacement due to the hat mode and L_k the length of barge k .)

Current Participants

ChevronTexaco

ConocoPhillips

Norsk Hydro

OTRC

Petrobras/USP

Shell

Statoil

Appendices (available at <http://www.wamit.com/publications.htm>)

Numerical studies of directional wavemaker performance, by J.F. Odea and J.N. Newman
(28th American Towing Tank Conference)

Trapping of water waves by moored bodies, by J.N. Newman
(To appear in J. of Engineering Mathematics)