Linear motions of fish tanks

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1 Introduction

Floating tanks used for fish farms consist of fluid basins which are separated from the exterior domain by thin boundary surfaces. Waves in the exterior domain excite motions of the tank, which generate sloshing in the interior. Since the total mass consists almost entirely of the interior fluid, strong coupling exists between the sloshing and the motions of the tank. This is an important feature of these structures, compared to ships with interior tanks where the effect of sloshing on the body motions is relatively small. The possibility of violent sloshing at natural frequencies has motivated nonlinear analyses [1,2]. However the sloshing amplitude is reduced substantially if the tank is freely floating. If the sloshing is moderate it is logical to use a linear analysis, which is useful to study parametric variations such as the shape of the tank.

Computations are presented here for the surge, pitch and sloshing motions of circular cylindrical tanks with vertical axes, floating on the free surface in a fluid of infinite depth. Heave is neglected since it causes a uniform vertical motion of the fluid in the tank which has no effect on the sloshing or other modes. The cylinders have unit radius. Results are presented for four values of the depth (draft) d, as shown in Figure 1, to illustrate the dependence on depth. The domain interior to the cylinders is occupied by fluid with the same density as in the exterior domain, with a free surface at the same height. The boundary separating the two domains is assumed to be rigid, with zero thickness. Thus there is no rigid-body mass or hydrostatic restoring moment. The origin is at the center of the interior free surface with z positive upwards. Only the first longitudinal sloshing mode is considered. The amplitude is measured by the elevation of the interior free surface near the downwave side of the cylinder, at the point $x = 0.9,\ y = 0.0$. The frequency is $\omega = \sqrt{gK}$ where K is the wavenumber in the exterior domain and g is the gravitational acceleration. The pitch angle is measured in radians.

The frequency-domain code WAMIT is used for the computations. The procedure for coupling the exterior and interior solutions is described in [3]. The higher-order option is used, with exact representation of the geometry and continuous B-spline representation of the unknown potential on the boundary surface.

In Section 2 the simpler problem is discussed where there is no exterior fluid and forced oscillatory motions are applied to the tank. Results are shown for the sloshing amplitude and also for the added-mass coefficients, which represent the pressure force and moment acting on the interior surface of the tank. These are helpful in explaining the results in Section 3 where the exterior domain is included and computations are presented for the body motions and sloshing caused by incident waves. Special cases are described in Section 2 where the sloshing due to pitch is non-resonant, based on the analysis in the Appendix.

2 Forced motions of the tank

Forced oscillatory surge and pitch motions with unit amplitude are applied to the tank. Only the interior fluid domain is considered. There is no damping from wave radiation and no hydrostatic restoring force or moment; thus the pressure force and moment can be expressed in terms of the added-mass coefficients shown by the solid lines in Figure 1. The sloshing amplitudes are shown by dashed lines. Except for the special case described in the next paragraph the results in Figure 1 are all singular when the wavenumber k based on the tank depth is equal to the first positive zero of $J_1'(k)$, the derivative of the Bessel function of order one. The value of this zero, $k_1 = 1.841184...$, is the wavenumber of the first natural mode. The corresponding values of $K_1 = k_1 \tanh k_1 d$ are shown for each cylinder in Figure 1. Resonance occurs with unbounded amplitude if $K = K_1$.

There is no resonance in the pitch mode for the cylinder with depth $d = 0.71528$. This is due to cancellation from the side and bottom where the normal velocity has opposite signs. This special case was discovered from the numerical results, and confirmed by the analysis in the Appendix. From equation (9) it follows that the amplitude of the n'th natural mode is non-singular for pitch or roll if

$$k_n d = \cosh^{-1}(2) = 1.316958\ldots. \tag{1}$$

For the first natural mode the depth is $d = 1.316958/1.841184 = 0.715278$. For this case $A_{55} \approx 0.17$ and $|\zeta_5| < 0.1$, except very close to $K = K_1$ where somewhat larger values of order $10^{-5} \times \zeta_1$ are computed. Thus

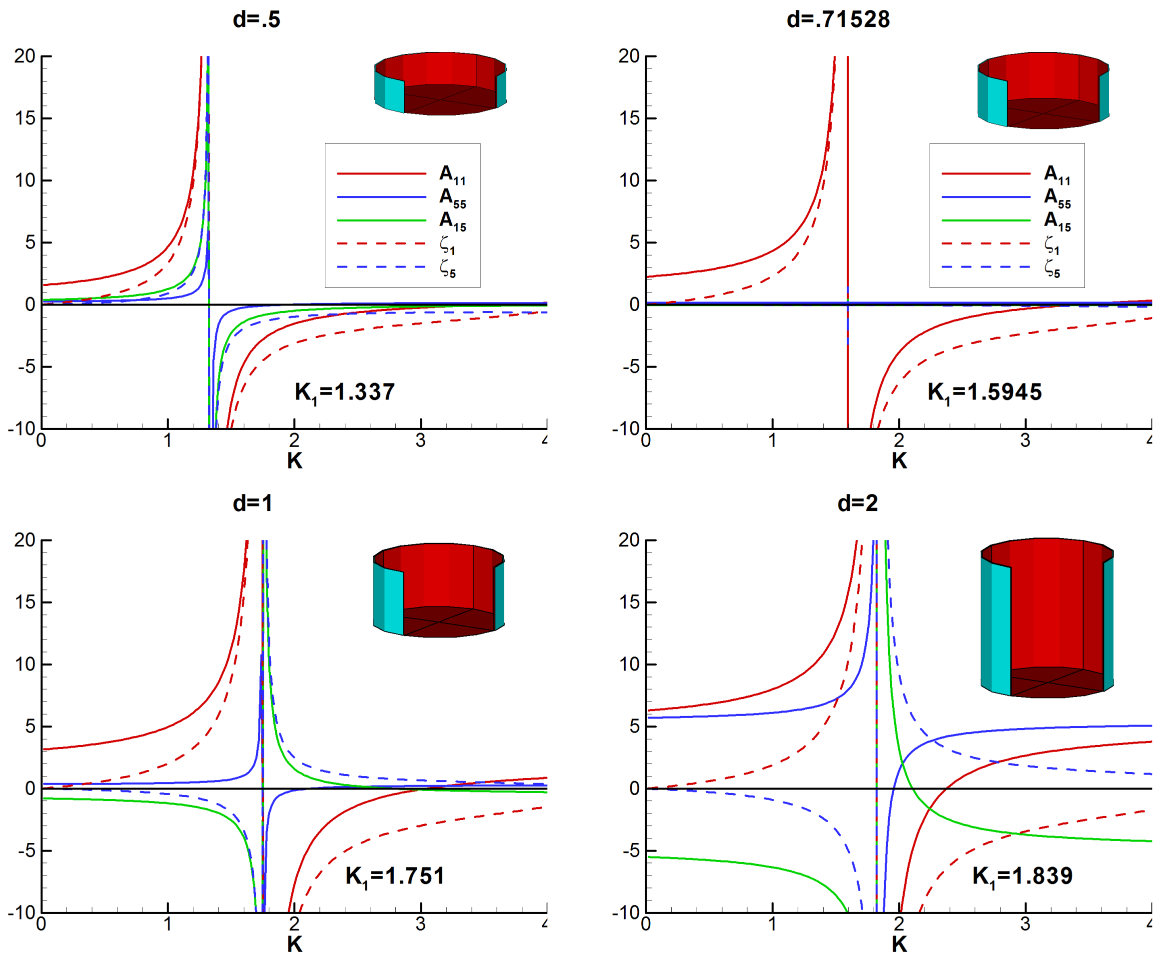

Figure 1: Added-mass coefficients (A_{ij}) and sloshing amplitudes (ζ_i) for forced oscillatory motion of the tank with no exterior fluid domain. The mode index $i = 1$ for surge and $i = 5$ for pitch. The value of the resonant wavenumber K_1 is shown for each tank. In the perspective figures one quadrant of the side is removed to show the interior.

the motions due to pitch are nonzero but negligible compared to surge. Similar results have been confirmed for the higher-order modes $k_2 = 5.33144$, $k_3 = 8.53632$ and corresponding depths $d = 0.247018$, $d = 0.154277$.

The case $d = 0.71528$ represents a transition for the results shown in Figure 1. For $d = 0.5$ all of the added-mass and sloshing values are positive if $K < K_1$ and negative if $K > K_1$. For $d = 1$ and $d = 2$ the signs of A_{15} and ζ_5 are reversed for $K < K_1$, and also near the singularity for $K > K_1$. These properties have been confirmed for other values of the depth, even close to $d = 0.71525$. Similar results have been computed for hemispheroids, where the hemisphere $d = 1$ is the obvious special case with no sloshing caused by pitch.

3 Free motions in waves

Figure 2 shows the response-amplitude operators (RAOs) of the rigid-body motions and sloshing amplitude if the tank is freely floating in the exterior domain, with incident plane waves propagating in the $+x$-direction with unit amplitude and wavenumber K. These results include the case with coupled surge and pitch, and also the cases with only one degree of freedom in either surge or pitch.

The case where surge is the only mode is described in [4]. The RAO ξ_1 is equal to the ratio of the exciting and restoring forces. The restoring force, which includes the tank added mass, is infinite at $K = K_1$. Thus $\xi_1 = 0$ at $K = K_1$, as shown by the blue lines in Figure 2. Resonance occurs when the negative added mass of the tank is cancelled by the positive external added mass, or the total added mass is zero. For the cylinder $d = 2$ this zero is close to $K = K_1$, where the tank added mass increases rapidly, resulting in a highly-tuned resonance at $K = 2.22$. For the other cylinders the zero occurs at a larger wavenumber, with a more gradual change of the added mass. The analogous case where pitch is the only mode is shown by the green lines in

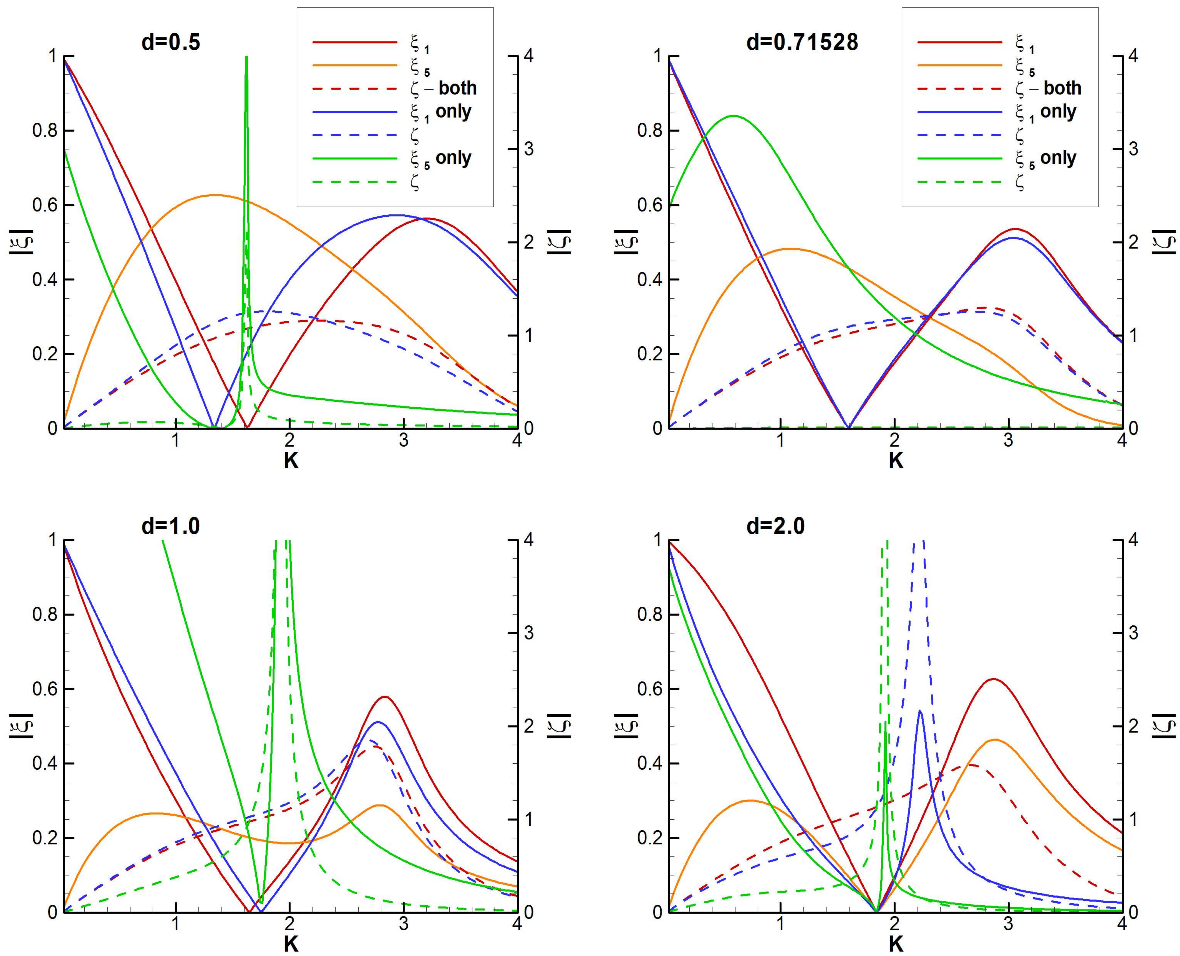

Figure 2: Free motions in waves and sloshing amplitude. The RAOs $|\xi_i|$ are shown by the solid lines and measured by the left axis. The sloshing amplitude $|\zeta|$ is shown by the dashed lines and the right axis.

Figure 2. As for surge, the pitch RAO ξ_5 is zero at $K = K_1$, but unlike surge there is a sharp resonant peak at a slightly greater wavenumber for all depths d. This is caused by the more rapid increase of A_{55} from $-\infty$ toward zero, cancelling the positive external added moment of inertia at a wavenumber closer to K_1 than in the case of surge. The depth 0.71525 is an exception, since there is no tank resonance generated by pitch.

The red and yellow lines in Figure 2 show the results when both modes are free. Coupling with pitch has a relatively small effect on the maximum amplitudes of surge and sloshing for $d = 0.5$ and 0.71525. The principal change for $d = 0.5$ is to move the zero of surge to a larger wavenumber. This is different from the situation with a single degree of freedom described above, where the RAO is zero at $K = K_1$. Instead ξ_1 and ξ_5 are nonzero at $K = K_1$, but 180° out of phase with relative magnitudes such that the sloshing resonances are cancelled. The same cancellation occurs for $d = 1$ and $d = 2$, except that the 180° phase difference is in the sloshing amplitudes instead of the RAOs. For $d = 2$ the zeros in Figure 2 appear to be coincident at $K = K_1$, but in fact they are distinct; ξ_1 and ξ_5 are small but nonzero at $K_1 = 1.839$, with their zeros at 1.841 and 1.845 respectively. The cylinder $d = 0.71528$ is an exception, due to the absence of pitch resonance, and the zeros of the red and blue lines are coincident at $K = K_1$. With both degrees of freedom the maximum sloshing amplitude is 1.8 for $d = 1$, and less for the other depths.

If the tank is free in both modes their phases are identical (modulo 180°). This is a general property of axisymmetric bodies, including bodies with internal tanks if there is no damping in the internal domain. The phases are practically constant, with values close to $\pm 90°$ up to the wavenumbers where the RAOs are zero.

In the low-frequency limit $K \to 0$ the fluid in the tank is 'frozen' in the surge mode, moving in uniform translation with the body. Thus $\xi_1 \to 1$ as $K \to 0$, in the same manner as a conventional rigid body. This limit is applicable both in the case where surge is the only mode, and where it is coupled with pitch. Since $\xi_5 = 0$ in the former case, and the equation of motion for the surge force holds in both cases, it follows that $\xi_5 \to 0$ when both modes are free. Conversely, if pitch is the only mode it is equal to the ratio of the exciting moment and restoring moment; since both are proportional to $\omega^2 = gK$ for $K << 1$, ξ_5 tends to a nonzero limit if it is the only mode. These limits are confirmed in Figure 2.

The tanks considered here have no hydrostatic restoring moments. Practical structures must have sufficient positive buoyancy provided by finite thickness of the tank walls and/or external floats. Thus a resonant pitch mode will exist at a relatively low frequency and wavenumber, proportional to the restoring moment. This resonance may be highly tuned, with a singular effect on the results in Figure 2.

References

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[3] Newman, J.N. 'Wave effects on vessels with internal tanks,' 20th IWWWFB (2005).

[4] Faltinsen, O.M. & Timokha, A.N. 'Sloshing,' Cambridge University Press (2009).

Appendix – Non-resonant sloshing due to pitch

Following [4], Section 5.4.4, with $x = r\cos\theta$, the potential can be expressed in the form $\phi(r,z)\cos\theta$ where

$$\phi(r,z) = F(r,z) + \sum_{n=1}^{\infty} c_n \frac{J_1(k_n r)}{J_1(k_n)} \cosh(k_n(z+d)), \tag{2}$$

and

$$F(r,z) = rz - \sum_{n=1}^{\infty} s_n \frac{J_1(k_n r)}{J_1(k_n)} \frac{\sinh(k_n(z+d/2))}{\cosh(k_n d/2)}. \tag{3}$$

The function F, known as the Stokes-Joukowski potential, satisfies the boundary conditions $F_r = z$ on $r = 1$ and $F_z = -r$ on $z = -d$ and $z = 0$. From the latter conditions it follows that

$$-r = r - \sum_{n=1}^{\infty} s_n k_n \frac{J_1(k_n r)}{J_1(k_n)}. \tag{4}$$

The coefficients s_n can be evaluated to satisfy (4) but that is not necessary here. Instead, using (4) to replace r in the first term on the right side of (3), and combining this result with (2),

$$\phi(r,z) = \sum_{n=1}^{\infty} \frac{J_1(k_n r)}{J_1(k_n)} \left[\tfrac{1}{2} s_n k_n z - \frac{s_n \sinh(k_n(z+d/2))}{\cosh(k_n d/2)} + c_n \cosh(k_n(z+d)) \right]. \tag{5}$$

The coefficients c_n are determined from the free-surface boundary condition $K\phi - \phi_z = 0$ on $z = 0$. Thus

$$\sum_{n=1}^{\infty} \left[\tfrac{1}{2} s_n k_n - s_n K \tanh(k_n d/2) + c_n K \cosh(k_n d) - c_n k_n \sinh(k_n d) \right] \frac{J_1(k_n r)}{J_1(k_n)} = 0. \tag{6}$$

From the orthogonality of the Bessel functions it follows that

$$\frac{c_n}{s_n} = \frac{K \tanh(k_n d/2) - \frac{1}{2} k_n}{K \cosh(k_n d) - k_n \sinh(k_n d)}. \tag{7}$$

At resonance $K = k_n \tanh(k_n d)$ and the denominator of (7) is zero. Thus c_n is finite if and only if the numerator is zero, or

$$1 = 2(K/k_n)\tanh(k_n d/2) = 2\tanh(k_n d)\tanh(k_n d/2). \tag{8}$$

After using elementary relations for the hyperbolic functions it follows that

$$\cosh(k_n d) = 2. \tag{9}$$

If the center of rotation is at $z = z_c$, the term rz in (3) is replaced by $r(z - z_c)$, and (9) is replaced by $\cosh(k_n d) = 2/(1 + K z_c)$. Thus, for any depth d, the singularities due to surge and pitch cancel if the tank is rotated about the point

$$z_c = (2\,\mathrm{sech} k_n d - 1)/K. \tag{10}$$

The same results (8-10) can be derived for any cylindrical tank where the geometry conforms to separable coordinates of the Laplace equation. The only changes required are to replace the Bessel and trigonometric functions by the appropriate separable solutions in the horizontal plane, and use the corresponding natural wavenumbers k_n. Thus (1) and (8-10) are valid for a variety of cylindrical shapes, including rectangular tanks.