Wave effects on hinged bodies
Part II – hinge loads

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1 INTRODUCTION

In Part 1 the methodology is explained for using WAMIT to analyse vertical motions of a hinged array consisting of N rigid modules. The deflections of the hinges are represented by an appropriate set of generalized modes which are either symmetric or antisymmetric about the plane $x = 0$. This complicates the definition of the modes, but leads to a substantial reduction in the computational cost for an array which has one or two planes of symmetry. Computations are presented for an illustrative example consisting of a rectangular articulated barge with five modules arranged in a longitudinal array with hinged joints. Each module has a length of 1200 feet, beam 500 feet, and draft 20 feet. Vertical motions at the stern, hinge joints, and bow are computed for a range of wave periods and heading angles.

As noted in Part 1, the structural analysis of loads on each module can be performed in an efficient manner using additional generalized modes to represent the loads and/or deflection of each module. We shall illustrate this procedure here by computing the shear loads acting on each hinge. The appropriate modes are derived in Section 2. In Section 3 the analysis leading up to the evaluation of the loads is presented. In Section 4 illustrative computations are presented for the same array geometry as in Part 1. To be consistent with the notation of Part 1, the heave/pitch modes will be denoted by the indices $j = 1, 2$, the hinge modes by $j = 3, 4, ..., N+1$, and the extra modes introduced for the evaluation of structural loads by $j = N+2, N+3, ...$.

Following the same notation as in Part 1, the origin is at the center of the array with hinges at $x = x_n$ $(n = 1, 2, ..., N-1)$, numbered from the stern to the bow in the direction of increasing x. A nondimensional coordinate $u = x/L$ is also used, where L is the length of each module, and $u = u_n$ denotes the corresponding hinge positions.

2 GENERALIZED SHEAR MODES

The vertical loads acting at the hinges can be computed in an efficient manner by defining additional generalized modes which are not used to represent actual modes of motion, but only to integrate the distributed loads on each module. Appropriate modes for this purpose are based on the Heaviside step-function defined by $H(x)$, where $H = 0$ if $x < 0$ and $H = 1$ if $x > 0$. The unsteady vertical load exerted on the hinge at $x = x_n$ can be evaluated by multiplying the inertial and pressure forces at each point on the structure by the mode

$$\hat{f}_j(u) = H(u_n - u) \qquad (j = N+2, , ..., 2N) \quad (n = 1, 2, 3, ..., N-1) \tag{2.1}$$

and integrating over the submerged surface.

Symmetric and anti-symmetric modes with the same discontinuity at $x = x_n < 0$ are readily defined by the relations

$$f_j(u) = H(u_n - u) + H(u - u_{N-n}) \qquad (j = 2[\tfrac{N}{2}] + 3, ..., 2N-1) \quad (n = 1, 2, ..., [\tfrac{N-1}{2}]) \tag{2.2}$$

$$f_j(u) = H(u_n - u) - H(u - u_{N-n}) \qquad (j = 2[\tfrac{N+1}{2}] + 2, ..., 2N) \quad (n = 1, 2, ..., [\tfrac{N}{2}]) \tag{2.3}$$

where the index j is odd or even, respectively, starting at $j = N+2$. When N is odd the number of symmetric (2.2) and antisymmetric (2.3) modes is the same, $(N-1)/2$. When N is even there are $N/2 - 1$ symmetric modes (2.2) and $N/2$ antisymmetric modes (2.3); the 'extra' antisymmetric mode jumps from +1 to -1 at the center hinge.

Figure 1 shows sketches of the modes described above, for the cases $N = 1, 2, 3, 4$.

The original step-function modes (2.1) can be recovered from (2.2-3) for $x_n < 0$ using the equation

$$\hat{f}_j(u) = \tfrac{1}{2}\left(f_{2j-N-2}(u) + f_{2j-N-1}(u)\right) \qquad (j = N+2, ..., [\tfrac{3N+1}{2}]) \quad (n = 1, 2, ..., [\tfrac{N-1}{2}]) \tag{2.4}$$

and, for $x_n = 0$ (N even), from the special equation

$$\hat{f}_j(u) = \tfrac{1}{2}\left(1 + f_j(u)\right) \qquad (j = [\tfrac{3N}{2}] + 1) \quad (n = [\tfrac{N}{2}]) \tag{2.5}$$

The analogous relations for the hinges $x = x_n > 0$ are complicated by the fact that the step function $H(x)$ is not symmetric. For this reason the appropriate relations are as follows:

$$\hat{f}_j(u) = 1 - \tfrac{1}{2}\left(f_{5N-2j+2}(u) - f_{5N-2j+3}(u)\right) \qquad (j = [\tfrac{3N+1}{2}] + 1, ..., 2N) \quad (n = [\tfrac{N+1}{2}], ..., N-1) \tag{2.6}$$

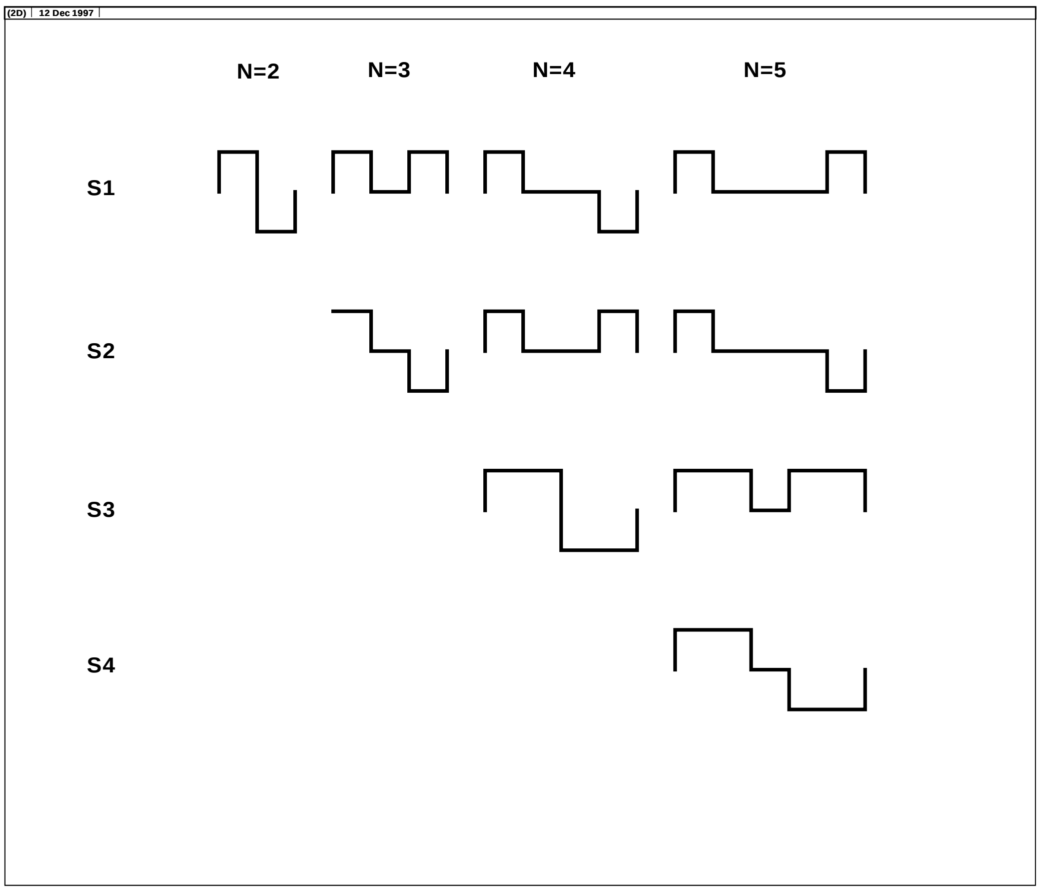

Figure 1: Modes used to represent the hinge shear loads. The number of separate modules N is shown at the top of each column, and the mode index Sn shown in the left column corresponds to the hinge index n.

The use of these modes to compute the hinge shear loads will be illustrated below.

3 SHEAR LOADS ON THE HINGES

Following the analysis described in Part 1, the (complex) amplitudes ξ_j of the free modes of motion are evaluated from the equations of motion

$$\sum_{j=1}^{N+1} \xi_j \left[-\omega^2(a_{ij} + M_{ij}) + i\omega b_{ij} + c_{ij}\right] = X_i \quad (i = 1, 2, 3, ..., N+1) \tag{3.1}$$

Here the summation on the left side is over all active modes including heave, pitch, and the hinge deflections. The hydrodynamic coefficients in (3.1) include the added mass a_{ij}, damping b_{ij} and exciting force X_i, M_{ij} is the inertia matrix associated with the body mass, and c_{ij} is the hydrostatic restoring matrix.

After solving the linear system of equations (3.1) for the $N+1$ free-mode amplitudes ξ_j, the generalized load corresponding to one of the fixed modes f_j $(j \geq N+2)$ can be evaluated in the form

$$F_i = -\sum_{j=1}^{N+1} \xi_j \left[-\omega^2(a_{ij} + M_{ij}) + i\omega b_{ij} + c_{ij}\right] + X_i \tag{3.2}$$

Using these formulae together with the discontinuous modes defined in Section 2, the vertical shear load acting on the hinge at $x = x_n$ can be evaluated from the following equations:

$$V_n = \tfrac{1}{2}\big(F_{2j-N-2}(u) + F_{2j-N-1}(u)\big) \qquad \begin{array}{l}(j = N+2, ..., [\frac{3N+1}{2}]) \\ (n = 1, 2, ..., [\frac{N-1}{2}])\end{array} \tag{3.3}$$

$$V_n = \tfrac{1}{2}(1 + F_j) \qquad \begin{array}{l}(j = [\frac{3N}{2}] + 1) \\ (n = [\frac{N}{2}])\end{array} \tag{3.4}$$

$$V_n = -\tfrac{1}{2}\big(F_{5N-2j+2} - F_{5N-2j+3}\big) \qquad \begin{array}{l}(j = [\frac{3N+1}{2}] + 1, ..., 2N) \\ (n = [\frac{N+1}{2}], ..., N-1)\end{array} \tag{3.5}$$

Note that the contribution from the first term on the right side of (2.6) vanishes in (3.5), due to the fact that the (heave) equation of motion is satisfied.

The evaluation of (3.2) requires specification of the inertia matrix M_{ij}. For an array of rectangular barges with uniform mass distribution the inertia matrix can be evaluated in terms of the generalized moments

$$I_{ij} = \int f_i(u) f_j(u) du \tag{3.6}$$

For the example to be treated below, the only non-zero moments which are required in (3.2) are readily evaluated by inspection. For the case $N = 4$ to be considered below, the required moments for $i = 7, 8, 9, 10$ are given as follows:

$$I_{71} = 2 \qquad I_{73} = 1$$
$$I_{82} = 4L \qquad I_{84} = 1$$
$$I_{91} = 4 \qquad I_{93} = 2 \qquad I_{95} = 1$$
$$I_{10,2} = 6L \qquad I_{10,4} = 2 \qquad I_{10,6} = 1$$

Here L is the length of each module.

Figures 2-3 show the computational results for the same array of barges considered in Part 1. The overall dimensions are 6000' long, 500' beam, and 20' draft, with five modules connected by hinges 1200' apart. The resilts shown in Figures 2-3 are based on computations with 1220 panels on one quadrant of the overall array. These loads are normalized by the product $\rho g A$, where ρ is the fluid density, g is gravity, and A is the incident wave amplitude.

To provide a basis for comparison, Figure 4 shows the total heave exciting force on the rigid array. This force is substantially larger, particularly in longer wave periods and for wave headings approaching the beam sea condition where the heave force is correlated along the length.

4 DISCUSSION

An efficient computational procedure has been described for the analysis of the vertical shear loads acting on the hinges of an articulated structure. Generalized shear modes are used to evaluate these loads. Instead of using the simpler Heaviside step function $H(x)$ to integrate the shear force in the conventional manner, symmetric and antisymmetric modes are defined which can be combined to give the same result. By including these modes in the hydrodynamic analysis the shear forces (3.2) can be evaluated directly from the standard set of WAMIT output parameters. As in the case of the hinge deflection modes defined in Part 1, the use of symmetric and antisymmetric generalized modes complicates the definition of the modes, but leads to a substantial reduction in the computational cost for an array which has one or two planes of symmetry. Our interest here is focussed on the shear force at the locations of the hinges, but the same procedure could be adopted (with suitably modified values of the coordinates $x = x_n$) to evaluate the shear load at other positions within each structural module.

The alternative and more conventional approach to evaluate the shear force is to integrate the vertical force acting on the structure, say starting at the stern and progressing toward the bow. While this is fundamentally straightforward, it does require more special post-processing of the WAMIT outputs. In this case the appropriate procedure would be to output the hydrodynamic pressure on each panel, multiply by the area and vertical component of the normal on

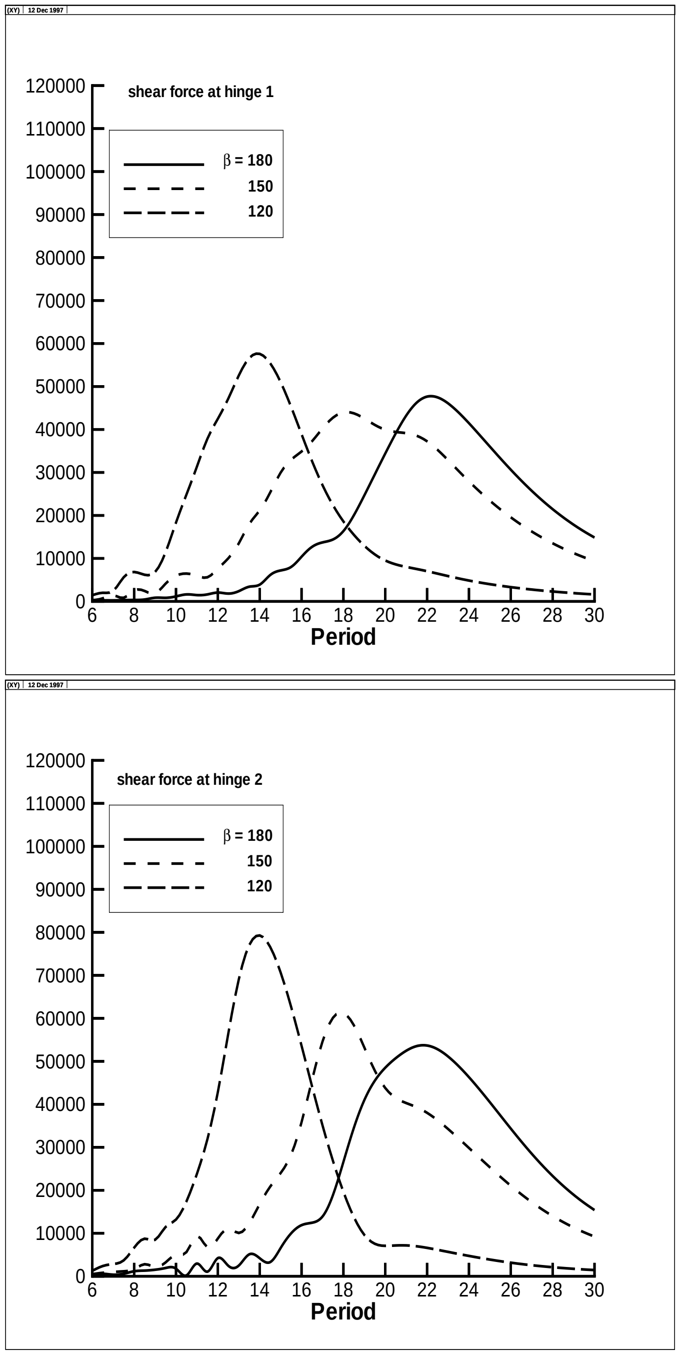

Figure 2: Shear loads on hinges 1 and 2.

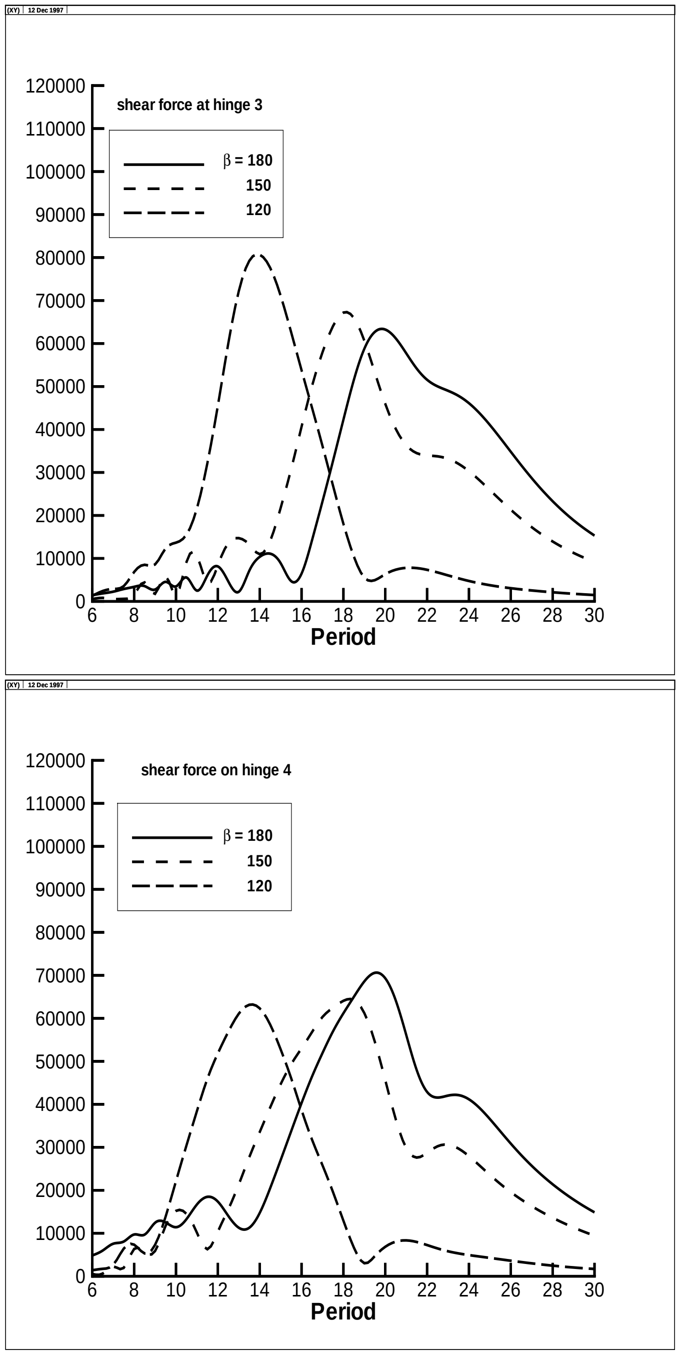

Figure 3: Shear loads on hinges 3 and 4.

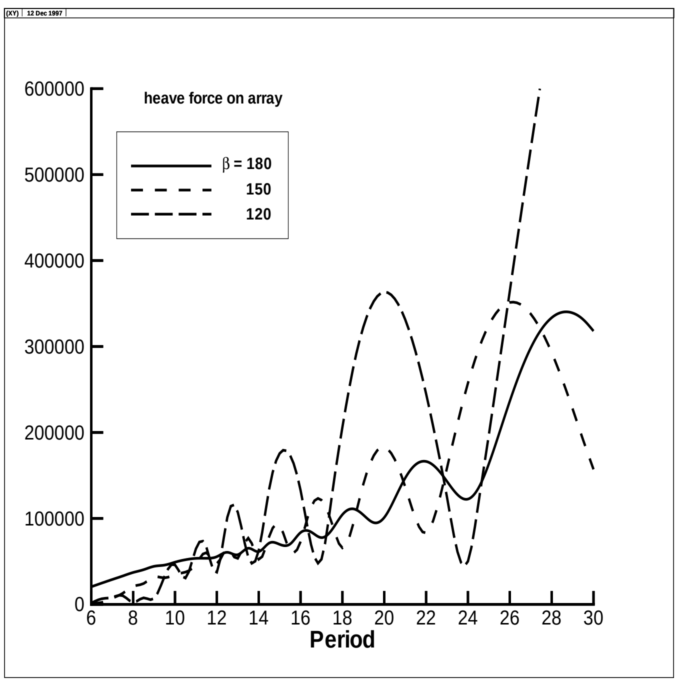

Figure 4: Heave exciting force on the complete (rigid) array.

each panel, and sum these products over the relevant area, say from the stern to hinge 1, then from hinge 1 to hinge 2, etc. In addition, the inertial load acting on each module and also the hydrostatic force must be evaluated and combined with the integrated pressure force. This procedure has been carried out for the same example described above, and the resulting 'direct' evaluations of the hinge shear loads are practically identical to those evaluated from the generalized modes.

One point should be noted in the context of using WAMIT to evaluate (3.2). Using the current (Version 5.3) or earilier versions it was necessary to output the separate components of (3.2) (complex amplitudes ξ_j of free motion, added mass, damping, restoring and exciting coeficients) and then to evaluate the sum (3.2) in a special post-processor. In the next release (Version 5.4pc) a new option can be used to output the complete force (3.2) directly. This will facilitate the evaluation of the hinge shear loads, and other structural loads, without extensive special post-processing.